

# Best Execution in Mortgage Secondary Markets

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### Abstract

A significant task faced by mortgage bankers attempting to profit from the mortgage secondary market is the best execution problem. This paper built a formal model to perform the best execution analysis. The model offers secondary marketing functionality including the loan-level best execution, guarantee fee buy-up/buy-down, and base/excess servicing fee. The model is formulated as a mixed integer programming problem. The case study shows that a realistic large-scale mixed-integer problem can be solved in an acceptable time (15 seconds) by CPLEX-90 solver on a PC.

**Keywords:** *best execution, secondary mortgage market, mortgage-backed security (MBS), Fannie Mae, MBS coupon rate, guarantee fee, guarantee fee buy-up/buy-down, servicing fee, mixed integer programming.*

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## 1. Introduction

Mortgage originators underwrite mortgages in the primary market. These mortgages bring fixed income cash flows to their owners. Besides keeping the mortgages as a part of the portfolio, a mortgage banker may sell the mortgages to other mortgage buyers or securitize the mortgages by pooling them into a mortgage-backed security (MBS) in the secondary market. The process that optimizes the mortgage assignments in the secondary market is known as the “Best Execution” strategy.

Three government-sponsored enterprises (GSEs) (Fannie Mae, Freddie Mae, and Ginnie Mae) provide different types of MBS swap programs in which mortgage bankers pool their mortgages into an appropriate MBS. Mortgages must meet certain standards to be eligible for pooling into a MBS. Usually, mortgage bankers prefer to pool mortgages into a MBS to get higher revenue. They sell mortgages as a whole loan only because those mortgages do not qualify for securitization. GSEs who issue the mortgage-backed securities (MBSs) provide insurance against the default risk and charge a guarantee fee.

This paper considers that each mortgage may be either sold as a whole loan or pooled into a MBS. We focus on the pass-through MBS swap programs provided by Fannie Mae. In particular, if a mortgage is sold as a whole loan, then the mortgage banker gets the price of the mortgage from the sale. On the other hand, if a mortgage is pooled into a MBS, the mortgage is securitized. The MBS investors, who buy the MBS, pay the price of the MBS to the mortgage banker. When a mortgage is pooled into a MBS, besides assigning the mortgage into the MBS with the proper coupon rate, mortgage bankers consider the guarantee fee buy-up/buy-down and mortgage servicing sell/retain features to maximize their total revenue.

We built a model to solve the best execution problem. The model is quite flexible and can be adjusted for different requirements of the mortgage securitization program. A case study shows that a realistic large-scale best execution problem can be solved in acceptable time by CPLEX-90 solver on a PC.

## 2. Problem description

Best execution is a central problem of mortgage secondary marketing. Mortgage bankers originate mortgages in the primary market and dispose of the mortgages in the secondary market to maximize their revenue. In the secondary market, each mortgage can be executed in two ways, either pooled into a MBS, or sold as a whole loan.

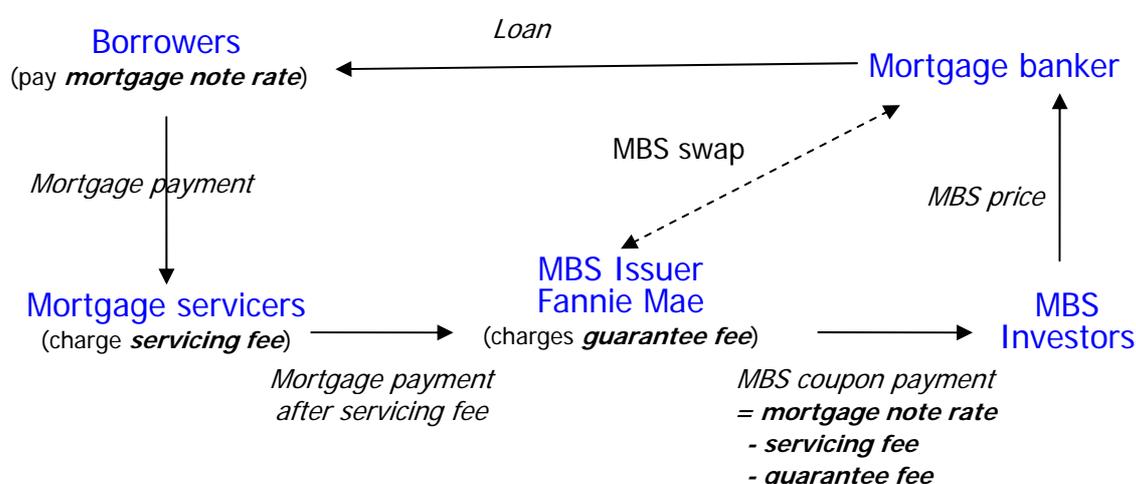
Mortgage bankers may sell mortgages at the price higher than the par value<sup>3</sup> and get revenue from

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<sup>3</sup> Mortgage bankers underwrite mortgages at a certain mortgage note rate. The par value is the value of the mortgage when the discount interest rate equals the mortgage note rate. In other word, the par value of a mortgage is its initial loan balance.

selling the mortgages. They may also pool the mortgages into MBSs. The participants in the MBS market can be categorized into five groups: borrowers, mortgage bankers, mortgage servicers, MBS insurers, and mortgage investors. The relationship and the cash flow between these five participants in the pass-through MBS market is shown in Figure 1.

**Figure 1: The participants and cash flows in the pass-through MBS market.** The mortgage borrowers pay a mortgage note rate. Mortgage servicers charge a servicing fee for the service. Fannie Mae charges a guarantee fee for the MBS insurance. Both the servicing fee and guarantee fee are defined as a percentage of the outstanding balance of mortgages. After these two fees, the pass-through MBS coupon rate received by investors equals “mortgage note rate – servicing fee – guarantee fee”.



Suppose mortgage bankers pool mortgages into a MBS. Borrowers pay the monthly payments in a fixed interest rate known as mortgage note rate. Mortgage servicers collect the monthly payments and forward the proceeds to the MBS investors. They obtain their revenue mainly from a servicing fee, which is a fixed percentage of the outstanding mortgage balance, and declines over time as the mortgage balance amortizes. Mortgage bankers may sell the mortgage servicing with a base servicing fee to the mortgage servicer and receive a payment. The bankers may choose to retain the servicing fee and provide the mortgage servicing. Fannie Mae, one of the government-sponsored enterprises that issue MBSs, provides MBS insurance, protecting the MBS investors against loss in the event of default of a borrower, and charges a guarantee fee.

The MBS investors pay the price of the MBS and get the MBS with the coupon rate equal to the mortgage note rate minus the sum of the servicing fee and the guarantee fee. Mortgage bankers get the payments from MBS investors. These payments can be used to originate more mortgages in the primary market.

## 2.1 The MBS program of Fannie Mae

In this section, we introduce the MBS products of Fannie Mae. See Jess Lederman (1997) for more resources about MBS programs.

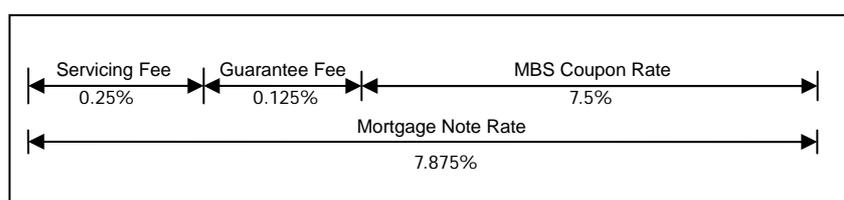
Fannie Mae purchases and swaps more than 50 types of mortgages on the basis of standard terms. This paper focuses on pass-through MBS swaps of 10-, 15-, 20- and 30-year fixed-rate mortgages. Mortgages must meet certain standards to be eligible for sale or swapping. Fannie Mae's MBS program has its own requirements, but provides flexibility in matching loan originations processing requirements. Mortgages must be pooled separately by the time to maturity. For instance, 30-year fixed-rate mortgages are separated from 15-year fixed-rate ones. The pass-through rate, or MBS coupon rate, generally trades on the half percent (4.5%, 5.0%, 5.5%, etc.) and mortgage lenders take advantage of pooling their loans to possible MBS coupon rate options. The mortgage note rate in each pool must support the pass-through rate plus minimum servicing fee plus the guarantee fee required by Fannie Mae. Therefore, the pass-through rate must satisfy the following equality:

$$\text{Mortgage note rate} = \text{Servicing Fee} + \text{Guarantee Fee} + \text{MBS coupon rate}.$$

### Guarantee fee

Mortgage bankers negotiate the guarantee fee with Fannie Mae. The guarantee fee is a fixed percentage (generally 25 base point (bp) to 35 bp based on the type of product) of the outstanding mortgage balance. Mortgage lenders have the opportunity to “buy down” or “buy up” the guarantee fee. “Buy down” means lenders reduce the spread required for the guarantee fee and pay an equivalent payment to Fannie Mae. On the other hand, “buy up” means lenders increase the spread of guarantee fee and receive a payment from Fannie Mae. For example, if a lender wants to include a 7.875% mortgage in a 7.5% pass-through MBS (Figure 2), he can buy down the guarantee fee to 0.125% from 0.25% by paying Fannie Mae the present value of equivalent of 0.125% difference and maintaining the 0.25% minimum servicing fee. If a lender chooses to include an 8.125% mortgage in the 7.5% pass-through MBS (Figure 3), instead of keeping the 0.375% servicing fee, he can sell the excess 0.125% servicing fee to Fannie Mae in return for a present value equivalent of 0.125%. The buy-down and buy-up guarantee fee features allow lenders to maximize the present worth of revenue.

**Figure 2: Guarantee fee buy-down.** A lender may include a 7.875% mortgage in a 7.5% pass-through MBS by buying down the guarantee fee to 0.125% from 0.25% and paying Fannie Mae the present value of equivalent of the 0.125% difference and maintaining the 0.25% minimum servicing fee.





When mortgages are assigned into a MBS, we consider the guarantee fees buy-up/buy-down feature and the servicing retain/sell feature in our model to maximize the total revenue. We formulate the best execution problem as a mixed integer programming problem.

### 3.1 Model description

The objective of the model is to maximize the revenue. Four sources of revenue are included in the model:

(1) *Revenue from mortgages pooled into a MBS or sold as a whole loan:*

$$\sum_{m=1}^M \left[ L^m \times \sum_{c=1}^{C^m} (P_c \times z_c^m) + P_{whole}^m \times z_w^m \right],$$

where

$$\begin{aligned} M &= \text{total number of mortgages,} \\ m &= \text{index of mortgages } (m = 1, 2, \dots, M), \\ L^m &= \text{loan amount of mortgage } m, \\ C^m &= \text{number of possible MBS coupon rates of mortgage } m, \\ c &= \text{index of MBS coupon rate,} \\ P_c &= \text{price of MBS with coupon rate index } c, \\ P_{whole}^m &= \text{price of mortgage } m \text{ that is sold as a whole loan,} \\ z_c^m &= \begin{cases} 1, & \text{if mortgage } m \text{ is pooled into MBS with coupon rate } c, \\ 0, & \text{otherwise,} \end{cases} \\ z_w^m &= \begin{cases} 1, & \text{if mortgage } m \text{ is sold as a whole loan,} \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Since each mortgage can be either pooled into a MBS or sold as a whole loan, we have the constraint

$$\sum_{c=1}^{C^m} z_c^m + z_w^m = 1. \quad (C1)$$

If mortgage  $m$  is pooled into a MBS with coupon rate index  $\hat{c}$ , then  $z_{\hat{c}}^m = 1$ ,  $z_c^m = 0$  for all

$c \neq \hat{c}$ , and  $z_w^m = 0$ , and the revenue from mortgage  $m$  equals  $L^m \times P_{\hat{c}} \times z_{\hat{c}}^m$ . On the other hand, if

mortgage  $m$  is sold as a whole loan, then  $z_c^m = 0$  for all  $c$ , and  $z_w^m = 1$ , and the revenue from mortgage  $m$  equals  $P_{whole}^m \times z_w^m$ . The total revenue is the summation of revenues from the  $M$

mortgages: 
$$\sum_{m=1}^M \left[ L^m \times \sum_{c=1}^{C^m} (P_c \times z_c^m) + P_{whole}^m \times z_w^m \right].$$

(2) Revenue from base servicing fee of mortgages pooled into a MBS:

$$\sum_{m=1}^M \left( L^m \times B^m \times z_{sbo}^m + L^m \times R_{sb}^m \times K_{sr}^m \times z_{sbr}^m \right),$$

where

$$\begin{aligned} B^m &= \text{base servicing value of mortgage } m, \\ z_{sbo}^m &= \begin{cases} 1, & \text{if the servicing of mortgage } m \text{ is sold out,} \\ 0, & \text{otherwise,} \end{cases} \\ R_{sb}^m &= \text{base servicing fee of mortgage } m, \\ K_{sr}^m &= \text{retained servicing multiplier of mortgage } m, \\ z_{sbr}^m &= \begin{cases} 1, & \text{if the base servicing of mortgage } m \text{ is retained,} \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Since the servicing of each mortgage can either be sold or retained, and the revenue from mortgage servicing exists only if the mortgage is pooled into a MBS instead of being sold as a whole loan, we impose the constraint

$$z_w^m + z_{sbo}^m + z_{sbr}^m = 1. \quad (C2)$$

If mortgage  $m$  is sold as a whole loan, then  $z_w^m = 1$ ,  $z_{sbo}^m = z_{sbr}^m = 0$ , and the revenue from servicing equals zero. On the other hand, if mortgage  $m$  is pooled into a MBS and the servicing of mortgage  $m$  is sold, then  $z_{sbo}^m = 1$ ,  $z_{sbr}^m = 0$ ,  $z_w^m = 1$ , and the revenue equals  $L^m \times B^m \times z_{sbo}^m$ ; otherwise  $z_{sbr}^m = 1$ ,  $z_{sbo}^m = 0$ ,  $z_w^m = 0$ , and the revenue equals  $L^m \times R_{sb}^m \times K_{sr}^m \times z_{sbr}^m$ . The total revenue from the  $M$  mortgages equals  $\sum_{m=1}^M \left( L^m \times B^m \times z_{sbo}^m + L^m \times R_{sb}^m \times K_{sr}^m \times z_{sbr}^m \right)$ .

(3) Revenue from excess servicing fee of mortgages pooled into a MBS:

$$\sum_{m=1}^M \left( L^m \times K_{sr}^m \times r_{ser}^m \right),$$

where

$$\begin{aligned} r_{ser}^m &= \text{retained excess servicing fee of mortgage } m, \\ K_{sr}^m &= \text{retained servicing fee multiplier of mortgage } m. \end{aligned}$$

If mortgage  $m$  is pooled into a MBS, the excess servicing fee generates revenue  $L^m \times K_{sr}^m \times r_{ser}^m$  from retaining the excess servicing fee. The total revenue from the  $M$  mortgages equals

$$\sum_{m=1}^M \left( L^m \times K_{sr}^m \times r_{ser}^m \right).$$

(4) Revenue from buy-up/buy-down guarantee fee of mortgages pooled into a MBS:

$$\sum_{m=1}^M (K_u^m \times L^m \times r_{gu}^m) - \sum_{m=1}^M (K_d^m \times L^m \times r_{gd}^m),$$

where

$r_{gu}^m$  = guarantee fee buy-up spread of mortgage  $m$ ,

$r_{gd}^m$  = guarantee fee buy-down spread of mortgage  $m$ ,

$K_u^m$  = guarantee fee buy-up multiplier of mortgage  $m$ ,

$K_d^m$  = guarantee fee buy-down multiplier of mortgage  $m$ .

Multipliers ( $K_u^m, K_d^m$ ) transfer the fixed income value of the guarantee fee to the present value.

Buying up the guarantee fee of mortgage  $m$  generates revenue,  $K_u^m \times L^m \times r_{gu}^m$ . On the other hand,

buying down the guarantee fee of mortgage  $m$  generates negative revenue (cost),  $K_d^m \times L^m \times r_{gd}^m$ . The

total revenue from the  $M$  mortgages equals  $\sum_{m=1}^M (K_u^m \times L^m \times r_{gu}^m) - \sum_{m=1}^M (K_d^m \times L^m \times r_{gd}^m)$ .

We consider the guarantee fee buy-up/buy-down and retain excess servicing fee only if mortgage  $m$

is pooled into a MBS. Therefore, when mortgage  $m$  is sold as a whole loan,  $r_{ser}^m$ ,  $r_{gu}^m$ , and  $r_{gd}^m$

should be zero. To define this condition, we impose the constraint

$$z_w^m + r_{gu}^m + r_{gd}^m + r_{ser}^m \leq 1. \quad (C3)$$

### Constraints

Besides constraints (C1), (C2), and (C3) listed above, the model contains other constraints. The most

important one is the balance equation for mortgage  $m$  pooled into a MBS:

$$\text{mortgage note rate} = \text{MBS coupon rate} + \text{servicing fee} + \text{guarantee fee}$$

We formulate this constraint as

$$\sum_{c=1}^{C^m} R_c z_c^m + r_{gu}^m - r_{gd}^m + r_{ser}^m \leq R_n^m - R_{sb}^m - R_{gb}^m,$$

where

$R_c$  = MBS coupon rate related to index  $c$ ,

$R_n^m$  = note rate of mortgage  $m$ ,

$R_{sb}^m$  = base servicing fee of mortgage  $m$ ,

$R_{gb}^m$  = base guarantee fee of mortgage  $m$ .

For all mortgages pooled into a MBS, the average excess servicing fee is limited by an upper bound  $R_{se}^a$ . This requirement is formulated as  $\sum_{m=1}^M L^m \times r_{se}^m \leq R_{se}^a \left( \sum_{m=1}^M L^m (1 - z_w^m) \right)$ . Besides the

aggregate level constraint, we categorize mortgages into groups by their *year to maturity* and consider the group-level average excess servicing fee limits  $R_{se}^j$  for each group  $j$  as

$\sum_{m \in j} L^m \times r_{se}^m \leq R_{se}^j \left( \sum_{m \in j} L^m (1 - z_w^m) \right)$ . The decision variables of the guarantee fee buy-up/buy-down

spreads and excess servicing fee are limited by their upper bounds  $0 \leq r_{gu}^m \leq R_{gu}^m$ ,  $0 \leq r_{gd}^m \leq R_{gd}^m$ , and

$0 \leq r_{ser}^m \leq R_{se}^m$ . Finally, we impose non-negativity constraints and binary constraints:  $r_{ser}^m, r_{gu}^m, r_{gd}^m \geq 0$ ,

$z_c^m, z_{whole}^m, z_{sbr}^m, z_{sbo}^m \in \{0,1\} \quad \forall m = 1, 2, \dots, C^m$

### 3.2 Model Formulation

This section summarizes the model described in the previous section. See the list of notations in the Appendix. The objective function and constraints in the model are linear and the model contains binary variables. Further, the best execution problem is formulated as a mixed integer programming problem.

	$\sum_{m=1}^M \left[ L^m \times \sum_{c=1}^{C^m} (P_c^m \times z_c^m) + P_{whole}^m \times z_w^m \right]$	$\left[ \begin{array}{l} \text{revenue from mortgages pooled into} \\ \text{a MBS or sold as a whole loan} \end{array} \right]$
<i>Max</i>	$+ \sum_{m=1}^M (L^m \times B^m \times z_{sbo}^m + L^m \times R_{sb}^m \times K_{sr}^m \times z_{sbr}^m)$	$\left[ \begin{array}{l} \text{revenue from base servicing fee of} \\ \text{mortgages pooled into a MBS} \end{array} \right]$
	$+ \sum_{m=1}^M (L^m \times K_{sr}^m \times r_{ser}^m)$	$\left[ \begin{array}{l} \text{revenue from excess servicing fee} \\ \text{of mortgages pooled into a MBS} \end{array} \right]$
	$+ \sum_{m=1}^M (K_u^m \times L^m \times r_{gu}^m) - \sum_{m=1}^M (K_d^m \times L^m \times r_{gd}^m)$	$\left[ \begin{array}{l} \text{revenue from buy-up/buy-down guarantee} \\ \text{fee of mortgages pooled into a MBS} \end{array} \right]$
<i>s.t.</i>	$\sum_{c=1}^{C^m} z_c^m + z_w^m = 1 \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{mortgage } m \text{ must be either pooled into MBS or} \\ \text{sold as a whole loan} \end{array} \right]$
	$z_w^m + z_{sbo}^m + z_{sbr}^m = 1 \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{if mortgage } m \text{ is sold, there is no servicing;} \\ \text{otherwise, servicing either be sold or retained} \end{array} \right]$
	$z_w^m + r_{gu}^m + r_{gd}^m + r_{ser}^m \leq 1 \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{if mortgage } m \text{ is sold, there is no guarantee fee} \\ \text{and excess servicing fee} \end{array} \right]$
	$\sum_{c=1}^{C^m} R_c z_c^m + r_{gu}^m - r_{gd}^m + r_{ser}^m \leq R_n^m - R_{sb}^m - R_{gb}^m \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{if mortgage } m \text{ is pooled into MBS, note rate=} \\ \text{MBS coupon rate+servicing fee+guarantee fee} \end{array} \right]$
	$\sum_{m=1}^M L^m \times r_{se}^m \leq R_{se}^a \left( \sum_{m=1}^M L^m (1 - z_w^m) \right)$	$\left[ \begin{array}{l} \text{the average excess servicing fee of all } M \\ \text{mortgages is limited by an upper bound.} \end{array} \right]$
	$\sum_{m \in j} L^m \times r_{se}^m \leq R_{se}^j \left( \sum_{m \in j} L^m (1 - z_w^m) \right) \quad \forall j = 1, 2, \dots, J$	$\left[ \begin{array}{l} \text{the average excess servicing fee of each MBS} \\ \text{group } j \text{ is limited by an upper bound} \end{array} \right]$
	$0 \leq r_{gu}^m \leq R_{gu}^m \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{for each mortgage } m, \text{ the guarantee fee buy-up} \\ \text{spread is limited by an upper bound} \end{array} \right]$
	$0 \leq r_{gd}^m \leq R_{gd}^m \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{for each mortgage } m, \text{ the guarantee fee} \\ \text{buy-down spread is limited by an upper bound} \end{array} \right]$
	$0 \leq r_{ser}^m \leq R_{se}^m \quad \forall m = 1, 2, \dots, M$	$\left[ \begin{array}{l} \text{for each mortgage } m, \text{ the excess servicing fee is} \\ \text{limited by an upper bound} \end{array} \right]$
	$z_c^m, z_{whole}^m, z_{sbr}^m, z_{sbo}^m \in \{0, 1\} \quad \forall m = 1, 2, \dots, C^m$	$\left[ \begin{array}{l} \text{binary constraints} \end{array} \right]$
	$r_{ser}^m, r_{gu}^m, r_{gd}^m \geq 0$	$\left[ \begin{array}{l} \text{non-negativity constraints} \end{array} \right]$

## 4. Case Study

The case study was conducted with the mortgage dataset provided by the Ohio Savings Bank.

### 4.1 Input data

Input data are classified as follows.

- Data on mortgages: (1) loan amount of each mortgage; (2) time to maturity of each mortgage; (3) note rate of each mortgage; and (4) possible MBS pools for each mortgage assignment.
- Data on MBS: (1) price of MBS (related to maturity and MBS coupon rate)
- Data on multipliers and parameters: (1) base servicing value; (2) guarantee fee buy-down/buy-up multiplier; (3) retain servicing multiplier; (4) upper bounds of guarantee fee buy-up and buy-down; (5) upper bounds of excess servicing fee in loan, group and aggregate levels.

#### 4.2 Problem size

The considered dataset:

- 4,355 mortgages.
- Four different times to maturity: 10, 15, 20, 30 years separating mortgages into four groups.
- 13 possible MBS coupon rates from 3.5% to 9.5% increasing in increments of 0.5%.

The problem size:

- 13,065 real variables.
- 66,190 binary variables.
- 30,490 constraints (binary and non-negativity constraints are not included).

#### 4.3 Solver

We used CPLEX-90 to solve the large-scale problem on an Intel Pentium 4, 2.8GHz PC. The problem was solved in 15 seconds with a solution gap<sup>4</sup> of less than 0.01%

#### 4.4 Solution

The total loan amount of the 4,355 mortgages is \$756,426,037. We assumed each mortgage is sold at the price of its loan amount. The total revenue equals the total loan amount (\$756,426,037). For the case when each mortgage either being pooled into a MBS or sold as a whole loan for the price of its loan amount, the optimal revenue is \$786,620,698, which is \$30,194,661 (4%) more than the total loan amount.

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<sup>4</sup> Solution gap defines a relative tolerance on the gap between the best integer objective and the object of the best node remaining. When the value  $|best\ node - best\ integer| / (|1e-10| + |best\ integer|)$  falls below this value, the MIP optimization is stopped.

**Table 1: Total revenue under different execution strategies**

Execution strategy	Total revenue
Mortgages are sold as a whole loan at the price of loan amount	\$756,426,037 (total loan amount)
Best execution strategy: mortgages are either pooled into a MBS or sold as a whole loan at the price of loan amount	\$786,620,698

If each mortgage is sold at the price of its loan amount, the total revenue equals \$756,426,037. With the best execution strategy such that each mortgage could be either pooled into a MBS or sold as a whole loan for the price of its loan amount, the revenue is improved to \$786,620,698, which is \$30,195,014 (4%) more than the total loan amount.

## 5. Conclusion

We built a formal model to perform the best execution analysis. The model includes the loan-level best execution for a MBS/whole loan, guarantee fee buy-up/buy-down, and base/excess servicing fee. The model is formulated as a mixed integer programming problem. The case study shows that a realistic large-scale mixed integer problem can be solved in acceptable time (15 seconds) by CPLEX-90 solver on a PC.

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## Appendix- Summary of notations for the model

The notation of the model is separated in three groups as follow.

### **Indices:**

$m$  = index of mortgages (1,2,...,M)

$M$  = total number of mortgages

$j$  = index of mortgage groups (1,2,...,J)

$J$  = total number of groups

$c$  = index of MBS coupon rate (1,2,...,C<sup>m</sup>)

$C^m$  = number of possible MBS coupon rates of mortgage  $m$

**Decision variables:**

$z_c^m$  = binary variable of mortgage  $m$  pooled into MBS with coupon rate index  $c$

$z_w^m$  = binary variable of mortgage  $m$  sold as a whole loan

$z_{sbo}^m$  = binary variable of the servicing of mortgage  $m$  is sold

$z_{sbr}^m$  = binary variable of the base servicing fee of mortgage  $m$  is retained

$r_{gu}^m$  = guarantee fee buy-up spreads of mortgage  $m$

$r_{gd}^m$  = guarantee fee buy-down spreads of mortgage  $m$

$r_{ser}^m$  = retained excess service fee of mortgage  $m$

**Input Data**

$L^m$  = loan amount of mortgage  $m$

$P_c$  = price of MBS with coupon rate index  $c$

$P_{whole}^m$  = price of mortgage  $m$  that is sold as a whole loan

$R_c$  = MBS coupon rate related to index  $c$

$K_u^m$  = guarantee fee buy-up multiplier of mortgage  $m$

$K_d^m$  = guarantee fee buy-down multiplier of mortgage  $m$

$R_n^m$  = note rate of mortgage  $m$

$R_{sb}^m$  = base servicing fee of mortgage  $m$

$R_{gu}^m$  = upper bound of buy-up guarantee fee of mortgage  $m$

$R_{gd}^m$  = upper bound of buy-down guarantee fee of mortgage  $m$

$R_{gb}$  = base guarantee fee of mortgage  $m$

$B^m$  = base servicing value of mortgage  $m$

$K_{sr}^m$  = retained servicing fee multiplier of mortgage  $m$

$R_{se}^m$  = upper bound of excess servicing fee of mortgage  $m$

$R_{se}^a$  = upper bound of average excess servicing fee of all mortgages pooled into MBSs

$R_{se}^j$  = upper bound of average excess servicing fee of mortgages pooled into MBSs in group  $j$