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Purpose

New methods of integrated risk modeling play an important role in determining the efficiency of bank portfolio management. We suggest a systematic approach for risk strategies formulation based on risk-return optimized portfolios, which applies different methodologies of risk measurement in the context of actual regulatory requirements.

Design/methodology/approach

We set up optimization problems to illustrate different levels of integrated bank portfolio management. We constrain economic capital allocation using different risk aggregation methodologies. We apply novel methods of financial engineering to relate actual bank capital regulations to recently developed methods of risk measurement (VaR and CVaR deviation). We run optimization problems with the Portfolio Safeguard (PSG) package by American Optimal Decision (www.AOrDA.com).

Findings

We find evidence that risk aggregation in ICAAP should be based on risk adjusted aggregation approaches, resulting in an efficient use of economic capital.

By using different values of confidence level α in VaR and CVaR deviation, it is possible to obtain optimal portfolios with similar properties. Before deciding to insert constraints on VaR or CVaR one should analyze properties of the dataset on which computation are based, with particular focus on the model for the tails of the distribution, as none of them is "better" than the other.

Research limitations/implications

This study should further be extended by an inclusion of simulation-based scenarios and copulae approaches for integrated risk measurements.

Originality/value

The suggested optimization models support a systematic generation of risk-return efficient target portfolios under the ICAAP. However, issues of practical implementation in risk aggregation and capital allocation still remain unsolved and require heuristic implementations.

Paper Classification

Research Paper

Keywords

Basel Capital Accord (Basel II), Value at Risk (VaR), Conditional Value at Risk (CVaR), Portfolio Optimization, Internal Capital Adequacy Assessment Process (ICAAP), Portfolio Safeguard (PSG).

1 Introduction

New regulations are imposing high standards on internal risk management in financial institutions. In its accord "International Convergence of Capital Measurement and Capital Standards – a Revised Framework" the Basel Committee on Banking Supervision has set forth a new set of regulations on risk management for financial institutions ("Basel II"). The new regulations are based on three pillars: pillar 1 consists of new minimum capital requirements, pillar 2 enforces qualitative standards on risk management, and pillar 3 requires risk management information disclosure, thus enforcing market discipline (Basel, 2004). While in recent years financial institutions have been focusing on the implementation of the quantitative requirements of pillar 1, attention recently shifted towards implementation of pillar 2 and pillars 3 requirements. Pillar 2 of the new Basel Accord postulates four principles of qualitative requirements on banks' internal risk management ("Supervisory Review Process"). The intention is to insure that banks have adequate capital to support all the risks in their ongoing business.

The first principle is also denoted as the "*Internal Capital Adequacy Process" (ICAAP)*: "Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels"[1]. To meet these requirements, the banks must aggregate material risks and allocate economic capital against them to cover potential financial losses. The internal capital adequacy needs to be balanced on an integrated portfolio level. A risk strategy must ensure capital adequacy in the overall business. While the quantitative standards of pillar 1 are clearly defined, many issues of internal risk management under pillar 2 methodologically are still unsolved or are difficult to implement, frequently due to the lack of sufficient data support. Banks are challenged to apply appropriate methodologies for integrated risk measurement under pillar 2.

We suggest an optimization model approach, which generates risk-return efficient portfolios with respect to internal (pillar 2) and regulatory (pillar 1) capital requirements, and illustrate its application by an example. For an optimized portfolio we derive capital allocation and an efficient risk strategy, which is required by ICAAP (for a summary refer to Rosenberg and Schuermann, 2004). One important issue in this context is to aggregate different risk types: market, credit, and operational risk, which show considerable variations in their distributions. Different approaches of measuring integrated risk and aggregating different risk types have been developed and have become a major object of discussion. We consider different approaches of risk aggregation in the context of portfolio optimization and ICAAP implementation. The capital requirements under pillar 1 thereby represent a minimum capital standard and a strict constraint which must be maintained continuously. Risk strategies frequently are derived by carrying forward actual portfolio compositions or heuristic allocations of economic capital or exposures by industry or branches. We suggest a systematic approach to derive consistent and efficient risk strategies from an optimal asset allocation.

The rest of the paper is organized as follows. Section 2 derives a general formulation of optimization models considering different approaches of risk aggregation and integrated risk measurement in ICAAP, section 3 describes the case study which applies the formulated optimization models, and section 4 concludes the paper.

2 Formulation of Optimization Model

2.1 Survey on Optimization Problem Statement

2.1.1 Related Research

Capital regulations can be considered in the context of portfolio optimization. In this context, the banks' objective function takes the form of an expected utility function, or alternatively, the banks face a Markowitz (1952) mean-variance portfolio-selection problem with additional constraints. For a discussion refer to Furlong and Keeley (1990), Kim and Santomero (1988), Koehn and Santomero (1980), Rochet (1992). Empirical research has been conducted to analyze the effects of regulations on the optimal capital structure in the interaction of regulation and bank management. Several studies were pursued in the option pricing framework. From the existing literature no clear statement can be deducted as to how capital regulations impact banks' risk strategies.

In our approach we apply novel methods of financial engineering to relate actual bank capital regulations to recently developed methods of risk measurement and portfolio optimization, as introduced by Rockafellar and Uryasev (2000). We extend the model for optimization of bank portfolios suggested by Theiler (2004) and apply the introduced risk-return management approach to an illustrative bank portfolio example. To optimize portfolios we use the Portfolio Safeguard (PSG) package by American Optimal Decisions (www.AOrDa.com).

2.1.2. General Problem Statement

We formulate a one period optimization model that maximizes expected returns of the bank portfolio with pillar 1 and pillar 2 requirements. Decision variables are the asset exposures. The capital constraints take into consideration economic and regulatory capital limits. Here is the general structure of the optimization problem for the bank portfolio [2]:

Objective function:	maximize expected returns
subject to constraints	
Constraint Set 1:	internal risk ≤ economic capital
Constraint Set 2:	regulatory risk ≤ regulatory capital
	constraints on the regulatory capital components (tier 1, 2, 3)
Constraint Set 3:	exposure constraints

2.2 Modeling Regulatory Requirements

2.2.1 Modeling Requirements on ICAAP (pillar 2)

To meet ICAAP requirements banks must measure relevant risks and allocate sufficient economic capital to cover them. The modeling of the internal capital adequacy raises a series of methodological questions about risk assessment, which financial institutions are implementing at different levels of sophistication. In the following we focus on actual questions of integrated risk measurement and risk aggregation.

Modeling interactions of different risks: risk integration approaches

The issue of risk aggregation has recently become an area of study [3]. A financial institution typically calculates the loss distributions for different risk types or for a number of business units on a standalone basis. Then, it aggregates the loss distributions and calculates the total economic capital for the whole enterprise (Hull 2007, p.373). In industry practice, easier-to-implement approximations are widely used, however more sophisticated approaches, such as copula-based methods, are increasingly discussed in financial theory.

Traditional approaches calculate different risks separately, without considering their interactions, and add risks to achieve an integrated risk measure. With this "*worst case approach*", total economic capital for n different risks is the sum of the economic capitals for each risk considered on a standalone basis (Hull 2007, p.374):

$$Tot\_Ec\_Cap = \sum_{i=1}^{n} E_i \quad . \qquad \text{eq. 1}$$

It can be observed that a simple adding up of risks (marginal distributions for individual contributors) significantly overestimates risks (and consequently economic capital), which is not surprising as it assumes the perfect interrisk correlation (Rosenberg and Schuermann, 2004).

The "normal distribution" approach is based on the assumption that loss distributions are normal (Hull, 2007. P.374). The standard deviation of the total loss from *n* sources then is:

$$\sigma_{total} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j\rho_{ij}} \;, \qquad \text{eq. 2}$$

where σ_i is the standard deviation of the loss from the *i-th* source of risk and ρ_{ij} is the correlation between risk i and risk j. In the parametric VaR approach the economic capital can be calculated by a multiple of the standard deviation of the normal distribution, i.e. the 99% VaR can be obtained by multiplying the portfolio standard deviation by 2.326. However, the joint normality assumption of risk factors imposes a distribution tail which is much thinner than the empirically observed one. "Normal" tails may significantly underestimate economic capital. This approach is not acceptable when one or more marginal densities exhibit significant negative skewness or excess kurtosis (Rosenberg and Schuermann, 2004). We do not consider further this approach in the context of ICAAP because the capital adequacy may not be maintained in this case.

In the "*hybrid approach*", risks are calculated on a standalone basis with possibly "heavy" tail distributions. Then, risks are aggregated by a correlation model, which combines marginal risks using the formula that would apply to an elliptical distribution [4]. Let $R=(\rho_{ij})$, i,j=1,...,n denote the correlation matrix, such that ρ_{ij} denotes the correlation of risk i and j. The overall risk then is calculated as the square root of the product of the vector of economic capitals, E = $(E_1,\ldots E_n)'$ with the correlation matrix R:

$$Tot\_Ec\_Cap = \sqrt{E'RE} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} E_i E_j \rho_{ij}} \quad . \qquad \text{eq. 3}$$

This approach is exactly correct for the special case when the marginal return distributions are normal. Rosenberg and Schuermann (2004) demonstrate that this approach may correctly aggregate heaviness in the tails of the individual loss distributions. It is surprisingly accurate and seems to combine copula based models quite well [5].

Integrated risk models represent the most advanced method of risk integration. Different approaches are suggested to derive the overall portfolio distribution. Copulae models aggregate the marginal loss risk distributions on the portfolio level. The essential idea of the copula approach is that a joint distribution can be factored into marginals and a dependence function called copula [6]. Simulation techniques are used at the portfolio level to derive the integrated portfolio loss risk distribution [7].

Our formulations of Constraints Set 1 (internal risk) in the optimization problems take into consideration different approaches of risk aggregation: worst case, hybrid and integrated approach.

Risk Measures for Integrated Risk Measurement

The ICAAP implementation needs appropriate risk measures on the integrated portfolio level. The discussion on risk measures has become a major object of research. The assumption of normally distributed returns, frequently assumed for market risk measurement, typically does not hold at the portfolio level, when different types of risk are aggregated. *Value-at-Risk* (VaR) risk measure is commonly applied in finance for market risk measurement. However, this measure has poor mathematical properties; in particular, it lacks sub-additivity if loss distributions are not normal. This means that portfolio diversification may increase risk, i.e., VaR of a combined portfolio may be higher than the sum of the VaRs of the sub-portfolios. VaR is a widely accepted risk measure and represents the industry standard of risk measurement. However, there is an increasing awareness of the problems which VaR may cause in risk measurement at the integrated portfolio level.

For a random variable X with continuous distribution function, *Conditional Value-at-Risk* ($CVaR_\alpha(X)$) equals the conditional expectation of X subject to $X \geq VaR_\alpha(X)$. The general definition of CVaR for random variables with a possibly discontinuous distribution function is more complex and can be found in Rockafellar and Uryasev (2002).

CVaR is sub-additive and is appropriate for risk measurement of any loss distribution [8]. In our case study we consider both VaR and CVaR in the context of portfolio optimization. We use *VaR and CVaR deviation measures* in internal risk constraint, as suggested by Rockafellar and Uryasev (2002, 2006b). These functions measure downside risk as the negative deviation from the expected outcome, which corresponds to economical concepts of risk and is commonly applied in banks risk management, where risk typically is defined as a potential for adverse deviation from expected results (Jorion, 2000, p. 81).

2.2.2 Modeling Requirements on Regulatory Capital (pillar 1)

According to the first pillar of the Basel II Accord, banks must meet a total risk-based capital ratio of eligible capital and regulatory risk [9]. The total capital ratio is defined as the eligible regulatory capital divided by the risk weighted assets. The total risk-weighted assets are determined by multiplying the capital requirements for market risk and operational risk by 12.5 (i.e., the reciprocal of the minimum capital ratio of 8%) and adding the resulting figures to the sum of risk-weighted assets for credit risk [10]. The capital ratio must be no lower than 8% [11].

The capital ratio is achieved by constraints on sub-portfolios of the bank book and the trading book and the risk is measured for these sub portfolios [12]. The bank needs to meet minimum capital requirements for the credit risk of the bank book, for the overall operational risk, and for the market risk of the trading book [13]. The different risk types are limited by qualifying capital components of the regulatory capital [14]. The tier 1 capital, i.e., the 'core' capital, includes common stockholders' equity. Tier 1 basically represents ownership value, which serves as the primary cushion of losses if the bank faces financial difficulties. Tier 2 capital comprises supplementary capital, such as long-term subordinate debt and loan loss reserves. Tier 3 capital consists of senior short-term debt. The maximum amounts of eligible tier 2 and tier 3 capitals are constrained with respect to the tier 1 capital available.

Pillar 1 capital rules include several constraints. First, the credit and operational risks are limited with respect to the maximum amount of tier 1 and tier 2 capitals. Second, the market risk of trading book is limited by the unused components of tier 1 and tier 2 capitals from the first constraint plus eligible tier 3 capital [15]. The use of tier 3 capital is limited against unused reserves of tier 1 and tier 2 capital [16] .

In optimization problems we include constraints on different risk types according to regulatory rules. We also define constraints on the use of different capital reserves.

2.3 Optimization Problem Formulation

2.3.1 Assumptions

The Basel II Accord allows different possibilities to model the capital requirements in pillar 1. For the implementation of *pillar 1* requirements, we make the following assumptions. For credit risk, we use the risk weights according to the Standardized Approach [17]. For market risk we apply the internal VaR model [18]. Operational risk is considered with the Basic Indicator

Approach. We treat operational risk as a constant, as it is not linked to the decision variables of our optimization models. Banks may use regulatory capital reserves under pillar 1 in a preference order to minimize funding costs on their capital reserves.

Under *pillar 2* we are using internal models based on bootstrapping of historical data for market and credit risk in the internal risk constraints [19]. For all other risks we are assuming a constant capital buffer, which is not linked to the decision variables.

2.3.2 Formulations of Problem Variations

As we focus on approaches of risk integration and integrated risk measurement in the ICAAP, we formulate different optimization problems with respect to internal risk measurement in the economic capital constraints (Constraint Set 1). According to the different approaches of risk aggregation, which we have presented in section 2.2, we achieve different problem variations.

At first we consider the "*worst case approach*". In practice, capital limits for different risk types frequently are fixed in a simple top-down approach for different risk types and standalone risks are measured against these limits. By allowing variable limits of the different sub-portfolios, we demonstrate how capital can be used more efficiently by allowing variation of limits between sub-portfolio risk types, while risk of sub-portfolios is measured on a standalone basis and the overall capital use is not changed. In optimization 2 we further include the hybrid approach and in optimization 3 we consider the integrated approach. With respect to the integrated approach, we consider the special case where return distributions can be derived from historical data, thus avoiding additional assumptions on the inter-relations of different types of risk, which might otherwise bias the optimal asset allocations between market and credit assets [20].

The following table summarizes the different approaches of risk integration which we consider in the problems' formulation.

Problem 1 **Worst Case Approach**	**Problem 2** **Hybrid Approach**	**Problem 3** **Integrated Approach**
Worst case aggregation with variable economic capital limits on market and credit portfolio	Correlation aggregation, economic capital on aggregated risk by correlation matrix	Integrated constraint, economic capital constraint on portfolio level

Table 1: Modelling the internal risk constraint (Constraint Set 1)

The considered optimization models differ only by the Constraint Set 1 for economic capital (ICAAP). Therefore, we formulate only Optimization Problem 1, for other problems we provide only alternative Constraint Set 1. We consider in each Constraint Set 1 two cases: a) VaR Deviation and b) CVaR Deviation for measuring risk.

Notations

We use the following notations in the Optimization Problems 1- 3:

Notation	Explanation
n_tb, n_bb	Number of assets in trading book and bank books, respectively
$n=n\_tb+n\_bb$	Total number of assets
$\mathbf{r}=(r_1,\ldots r_n)'$	Vector of estimated returns of assets
$x^{bb}=\begin{pmatrix} x_1^{bb} \\ \ldots \\ x_{n\_bb}^{bb} \end{pmatrix}$	(Sub-)Vector of decision variables of the bank book
$x^{tb}=\begin{pmatrix} x_1^{tb} \\ \ldots \\ x_{n\_tb}^{tb} \end{pmatrix}$	(Sub-)Vector of decision variables of the trading book
$\mathbf{x}=\begin{pmatrix} x_1 \\ \ldots \\ x_{n\_bb} \\ x_{n\_bb+1} \\ \ldots \\ x_n \end{pmatrix}=\begin{pmatrix} x_1^{bb} \\ \ldots \\ x_{n\_bb}^{bb} \\ x_1^{tb} \\ \ldots \\ x_{n\_tb}^{tb} \end{pmatrix}=\begin{pmatrix} \mathbf{x^{bb}} \\ \\ \mathbf{x^{tb}} \end{pmatrix}$	Vector of decision variables, i.e., asset exposures
$\begin{pmatrix} 1 & \ldots & \rho_{1m} \\ \ldots & \ldots & \ldots \\ \rho_{m1} & \ldots & 1 \end{pmatrix}$	Risk correlation matrix for sub-portfolios, j=1, …m.
$C_{ec\_other\_risk}$	Available economic capital for other risk (pillar 2)
$C_{ec\_total\_risk}$	Available economic capital for total risk (pillar 2)
$C_{reg\_tier\_i}$, $i=1,\ldots,3.$	Available Components of Regulatory Capital, tier i, i=1,…,3
reg_op_risk	Constant for regulatory capital for operational risk (Basic Indicator Approach)
$x_{ec\_cap}^{j}$, $j=1,\ldots, m$	Used economic capital in internal risk constraint for sub-portfolio j

x^a_i *i=1,...,3.*	Used regulatory tier i, i=1,…,3 capital
$w_j^{reg\_cr}$, *j=1,... n_bb*	Regulatory capital weights for credit assets of the bank book
$w_j^{reg\_sp}$, *j=1,... n_bb*	Regulatory risk weights for market risk constraint: specific risk of assets
$w^{reg\_mult\_mr}$	Regulatory multiplication factor for VaR model
α_int	Confidence level for internal economic risk constraints (pillar 2)
Total_Inv	Upper bound for overall investment exposures

Table 2: Notation in Optimization Problems

Optimization Problem 1 - <u>Worst Case Approach</u>

Maximize portfolio estimated return

$$max \sum_{i=1}^{n} r_i x_i \qquad \text{eq. 4}$$

subject to

Constraint Set 1 – Economic Capital (pillar 2)

Internal constraint on total economic capital

a) $$\sum_{j=1}^{m} VaR\_Dev_{\alpha\_\mathrm{int}}(x_j) \le C_{ec\_total\_risk} - C_{ec\_other\_risk}$$

b) $$\sum_{j=1}^{m} CVaR\_Dev_{\alpha\_\mathrm{int}}(x_j) \le C_{ec\_total\_risk} - C_{ec\_other\_risk} \qquad \text{eq. 5}$$

Constraint Set 2 – Regulatory Capital (pillar 1)

Balance equation for the regulatory capital covering credit risk

$$\sum_{i=1}^{n\_bb} w_i^{reg\_cr} x_i^{bb} + \sum_{i=1}^{n\_tb} w_i^{reg\_sp} x_i^{tb} + reg\_op\_risk = x_1^a + x_2^a \ . \qquad \text{eq. 6}$$

Balance equation for the regulatory capital covering market risk

$$12.5 * w^{reg\_mult\_mr} VaR_{99\%}(x^{tb}) \le x_3^a + (C_{tier\_1} - x_1^a) + (C_{tier\_2} - x_2^a) \ . \qquad \text{eq. 7}$$

Constraint limiting unused Tier-2 and used Tier-3 capital vs. unused Tier-1 capital

$$x_3^a + (C_{tier\_2} - x_2^a) \le 2.5(C_{tier\_1} - x_1^a) \ . \qquad \text{eq. 8}$$

Bounds on used Tier 1, Tier 2, and Tier 3 capital:

$$0 \le x_i^a \le C_{tier\_k} \quad i=1,\ 2,\ 3\ . \qquad \text{eq. 9}$$

Constraint Set 3 – Exposure Constraints

Upper and lower bounds on decision variables (exposures)

$$l_i \le x_i^{tb} \le u_i\ , \quad i=1,..,n\_tb\ , \qquad \text{eq. 10}$$

$$l_i \le x_i^{bb} \le u_i\ , \quad i=1,..,n\_bb\ .$$

Constant investment amount over all assets

$$\sum_{i=1}^{n\_bb} x_i^{bb} + \sum_{i=1}^{n\_tb} x_i^{tb} = Total\_Inv \quad . \qquad \text{eq. 11}$$

Further, we provide new variants of **Constraint Set 1** on internal risk (see Table 1) and keep the same objective function and all other constraints in Optimization Problems 2 and 3.

Optimization Problem 2 - <u>Hybrid Approach</u>

Constraint Set 1 – Economic Capital (pillar 2)

Internal constraint on sub-portfolios $\boldsymbol{x_j}$, j = 1, …, m:

$$\begin{aligned} &a) \quad VaR\_Dev_{\alpha\_int}(\mathbf{x}_j) \le x_{ec\_cap}^j\ , \\ &b) \quad CVaR\_Dev_{\alpha\_int}(\mathbf{x}_j) \le x_{ec\_cap}^j\ . \end{aligned} \qquad \text{eq. 5}$$

Internal constraint on total economic capital

$$\sqrt{(x_{ec\_cap}^1, \ldots x_{ec\_cap}^m) \begin{pmatrix} 1 & \ldots & \rho_{1m} \\ \ldots & \ldots & \ldots \\ \rho_{m1} & \ldots & 1 \end{pmatrix} \begin{pmatrix} x_{ec\_cap}^1 \\ \ldots \\ x_{ec\_cap}^m \end{pmatrix}} \le C_{ec\_total\_risk} - C_{ec\_other\_risk}\ . \qquad \text{eq. 6}$$

Optimization Problem 3 - Integrated Approach

Constraint Set 1 – Economic Capital (pillar 2)

Internal constraint on total economic capital

$$\begin{aligned} &a) \quad VaR\_Dev_{\alpha}(\mathbf{x}) \leq C_{ec\_total\_risk} - C_{ec\_other\_risk} \,, \\ &b) \quad CVaR\_Dev_{\alpha}(\mathbf{x}) \leq C_{ec\_total\_risk} - C_{ec\_other\_risk} \,. \end{aligned} \qquad \text{eq. 7}$$

3 Case Study

We used Portfolio Safeguard (PSG) to do the case studies. We posted MATLAB files to run these case studies in MATLAB environment on The MathWorks website (http://www.mathworks.com), in the file exchange-optimization area (search with the last name Serraino).

3.1 Assumptions and Setup

In our case study we illustrate different risk measurement methods in the context of ICAAP requirements. We optimize models, as described in section 2, to analyze effects of different methods of risk aggregation and measurement. Step 1 of the case study examines different risk aggregation approaches. Step 2 analyzes impact of different risk measures on aggregated risk measurement.

a) Portfolio Assets

We consider a typical bank book of a commercial bank. We use publicly available data (reports of the Federal Reserve Bank [21]) to determine the typical size and business assets of a large US commercial bank. We select four typical positions of bank assets: Securities

Government Bonds, Corporate and Industrial Credit, Real Estate Loans and Interbank Loans [22]. We re-scale the exposures to a balance sheet exposure, which corresponds to the average balance sheet exposure of the 5 largest US Commercial banks [23]. In addition we define typical market assets. We assume an exposure of the trading book of 20% of total balance sheet exposure and split it equally into the US and Euro market.

For building *loss distributions* we bootstrap historical index returns to avoid additional modeling assumptions and to facilitate the analysis of portfolio effects for different approaches of internal risk aggregation. The following table summarizes the portfolio setup:

	Expo-sure	Indices	Rating	Regula-tory cre-dit risk weight	Lower Ex-posure bounds	Upper Exposure Bounds
Setup credit portfolio	(bill. US $)				(in % of initial exposure)	
Securities Government debt	220	Markit iBoxx $ Domestic Sovereigns & Sub-Sov.AAA Total Return Index	AAA	0%	-20%	+20%
Commercial and Industrial Credit	130	Markit iBoxx $ Domestic Corporates BBB 1-3Y; Total Return Index	BBB	100%	-20%	+20%
Real Estate Loans	330	EPRA/NAREIT US Index	BB (ass.)	35%	-20%	+20%
Interbank Loans	40	Markit iBoxx $ Eurodollar Financials AA 1-3Y; Total Return Index	AA	20%	-20%	+20%
Setup market portfolio						
Equity Position USA	100	DJ inductrial average		2%	-100%	+100%
Equity Position Europe	100	DJ EURO STOXX 50		2%	-100%	+100%

Table 2: Summary of Portfolio Setup and Input Data

b) Assumptions on Constraint Set 1: Economic Capital

Risk modeling in ICAAP is frequently related to a one year time horizon, which corresponds to the accounting and capital planning periods. Accordingly, we choose a one year holding period for all assets.

We use time series for market and credit assets from January 3, 2000 to December 26, 2007. In the first constraint, we apply a 99% confidence level for VaR Deviation for the internal risk measurement.

c) Assumptions on Constraint Set 2: Regulatory Capital

For the regulatory constraints we apply approaches of the Basel rules for credit, market and operational risk as summarized in the section 2.2.2.

d) Further Assumptions

The upper bounds on available regulatory and economic capital are set equal to the corresponding capital use of the initial portfolio [24]. For the *exposure constraints* we assume that exposures in the banking book can be reduced or increased by 20%, while for the trading book they can be reduced or increased by 100% of the initial values. Additionally, we are assuming that the total investment capital does not exceed the total investment capital of the initial portfolio.

3.2 Step 1 of Case Study

Our objective is to examine the effects of different approaches of risk aggregation in the context of integrated portfolio optimization under ICAAP. We solve Problems 1–3, as described in section 2.3.2 with Constraint Set 1 (internal risk constraints), Constraint Set 2 and Constraint Set 3 (exposure constraints). We use VaR Deviation (case a) in the internal risk constraint.

Our first hypothesis is that from Problems 1 to 3 we will observe higher risk adjusted performance at the given level of economic capital, as portfolio effects are taken into consideration in a more risk-adjusted way: Problems 1 (standalone risk), Problems 2 (inter-correlation aggregation), and Problems 3 (integrated risk measurement).

We obtained the following optimal asset allocations in Problems 1-3:

		Initial Portfolio	**Problem 1**	**Problem 2**	**Problem 3**
Exposures (billion US $)			**Worst Case Approach**	**Hybrid Approach**	**Integrated Approach**
x_1^{bb}	Government Debt Securities	220	264	264	229.91
x_2^{bb}	Commercial and Industrial Credit	130	146.52	131.86	104
x_3^{bb}	Real Estate Loans	330	299.67	311.83	396
x_4^{bb}	Interbank Loans	40	48.00	48.00	32
x_1^{tb}	Equity Position USA	100	161.81	164.31	158.09
x_2^{tb}	Equity Position Europe	100	0	0	0
Total Exposure		**920**	**920**	**920**	**920**

Table 3: Exposures of Initial Portfolio and Optimal Solutions in Step 1

In order to analyze risk-return ratios for optimal portfolios we consider estimated returns and economic capital, which is allocated to cover the integrated risk with different approaches of risk aggregation. We define the portfolio risk-adjusted return on capital (RORAC) as the ratio of estimated return and economic capital of the portfolio [25].

Portfolio Risk-Return Ratios	**Initial Portfolio**	**Problem 1**	**Problem 2**	**Problem 3**
Economic Capital (billion US $)	129.44	129.44	129.44	129.44
Estimated Return (billion US $)	21.70	20.66	20.90	22.14
RORAC = (Est. Return) / (Econ. Capital)	16.76%	15.96%	16.15%	17.11%

Table 4: Risk-Return Ratios of Initial Portfolio and Optimal Solutions in Step 1

The optimized return is quite close for Portfolio 1 ($20.66 billion) and Portfolio 2 ($20.90billion), while for Portfolio 3($22.14billion) it is higher. This results in RORAC increasing from 15.96% (in optimal solution 1) to 17.11% (in optimal solution 3); the optimal portfolio 3 is the only one with higher return and RORAC than the initial portfolio. Thus, from the point of view of return adjusted for risk, the risk aggregation in Problem 3 allocated the economic capital in a more efficient way than the initial portfolio.

Summarizing, with our dataset we find evidence for our first hypothesis and conclude that the risk aggregation in ICAAP should be based on the risk-adjusted aggregation approach resulting in efficient use of economic capital. The diversification effects should be considered accurately.

3.3 Step 2 of Case Study

We consider the effect of using different risk measures in integrated risk assessment in the ICAAP. We follow Sarykalin, Serraino, and Uryasev (2008) in stating that one should not compare VaR and CVaR with the same confidence level, as they measure different parts of the distribution. In the considered dataset distributions of instruments are not very skewed, therefore there exists a confidence level α_1 such that the optimization of α_1-VaR is close to the optimization of α_2-CVaR in the sense that the objective values are close at optimality and the decision variables may have similar optimal positions. We find for the initial portfolio that

$$\alpha_2\text{-CVaR Deviation} \approx \alpha_1\text{-VaR Deviation}$$

when $\alpha_2 = 97.5\%$ and $\alpha_1 = 99\%$. Then we solve problems 1 to 3 replacing 99%- VaR Deviation in Constraint Set 1 with 97.5%-CVaR Deviation. Our hypothesis is that the solutions of these three problems with 97.5%-CVaR Deviation will be close to the solutions found with 99%-VaR Deviation.

Tables 7 and 8 show the results of Step 2.

(1)		(2) **Initial Portfolio**	(3) **Problem 1**	(4) **Problem 2**	(5) **Problem 3**
Exposures (billion US $)			**Worst Case Approach**	**Hybrid Approach**	**Integrated Approach**
x_1^{bb}	Securities Government Debt	220	264	264	227.73
x_2^{bb}	Commercial and Industrial Credit	130	133.43	119.74	104
x_3^{bb}	Real Estate Loans	330	396	321.88	391.17
x_4^{bb}	Interbank Loans	40	48	48	32
x_1^{tb}	Equity Position USA	100	78.57	166.37	165.10
x_2^{tb}	Equity Position Europe	100	0	0	0
	Total Exposure	**920**	**920**	**920**	**920**

Table 7: Exposures of Initial Portfolio and Optimal Solutions with 97.5% - CVaR Deviation in Step 2

Portfolio Risk-Return Ratios	**Initial Portfolio**	**Problem 1**	**Problem 2**	**Problem 3**
Economic Capital (billion US $)	129.44	129.44	129.44	129.44
Estimated Return (billion US $)	21.70	20.90	21.11	22.17
RORAC = Est. Return / Econ. Capital	16.76%	16.15%	16.31%	17.13%

Table 8: Risk-Return Ratios of Initial Portfolio and Optimal Solutions with 97.5% - CVaR Deviation in Step 2

Comparison of tables 5 and 6 with tables 7 and 8 show that the estimated return and RORAC with 99%-VaR Deviation in Step 1 and with 97.5%-CVaR Deviation in Step 2 are quite close for problem 3, while for problem 1 and 2 they are somewhat different and lead to a slightly higher RORAC for the CVaR deviation in step 2. While we recognize that these results are dependent on the dataset, we point out that by using different values of confidence level α in VaR and CVaR deviation it is possible to obtain optimal portfolios with similar properties. Thus one should analyze properties of the dataset on which computations are based, with particular focus on the model for the tails of the distribution, before deciding to insert constraints on VaR or CVaR, as none of them is "better" than the other. With PSG it is possible to impose simultaneously both VaR and CVaR constraints. We then estimate and compare CVaR Deviation of all optimal portfolios. Results are shown in table 9.

		Step 1			Step 2		
	Initial Portfolio	Problem 1	Problem 2	Problem 3	Problem 1	Problem 2	Problem 3
95%-CVaR Dev	49.29	24.70	27.52	42.44	23.40	29.86	43.33

Table 9: 95% CVaR Deviation of optimal portfolios.

In both Step 1 and 2 the 95% CVaR Deviation is higher for Problem 3 then for Problem 1 and Problem 2, but in each solution it is lower than 95% CVaR of the initial portfolio. This is an expected result because the risk aggregation model in Problem 3 is the least conservative. In order to implement the risk strategy for the ongoing business, capital limits must be allocated top-down to the different business units. For the capital allocation of the optimal target portfolio we suggest allocating capital according to the Euler allocation principle [26]. According to the Euler allocation principle, the risk contribution $r_j(x)$ of the j-th asset is based on the partial derivative of the overall risk measure $r(x)$:

$$r_j(\mathbf{x}) = \frac{\partial r(\mathbf{x})}{\partial x_j} x_j \ , \ j=1,\dots,n. \qquad \text{eq. 8}$$

PSG can calculate derivatives both for VaR and CVaR as well as many other risk measures. Capital allocations based on VaR-deviation for the optimal Portfolio 3 of Step 1 and for the initial portfolio are presented in Table 10.

	Euler Allocation Initial Portfolio	Euler Allocation Optimal Portfolio Problem 3 step 1
Securities Government Debt	-9.87	-8.90
Comm. and Industrial Credit	-1.71	-1.38
Real Estate Loans	74.16	99.29
Interbank Loans	-0.80	-0.56
Equity Position USA	25.32	32.24
Equity Position Euro	34.54	0
Total Capital [27]	121.64	120.69

Table 5: Capital Allocations based on VaR-deviation for the Initial portfolio and the optimal Portfolio 3 in Step 1

We observe that risk contributions of assets 1, 2 and 4 are negative, which corresponds to a positive diversification effect of these assets in the portfolio. The main risk driver in the portfolio is the Real Estate Loan position, which mirrors to some extent the actual credit crisis in financial markets. For the capital allocation, negative capital amounts lack meaningful interpretation. The issue of how to handle these effects of capital allocation is approached differently and not uniquely solved. The worst case allocation sometimes is suggested as an alternative allocation method; however, this allocation approach is not efficient, as was discussed in Step 1 above. Summarizing, we conclude that the suggested optimization model, as described in section 2 and applied in this case study, supports a systematic generation of risk-return efficient target portfolios under the ICAAP. However, issues of practical implementation in risk aggregation and capital allocation still remain unsolved and require heuristic implementations.

4 Conclusion

We suggested an optimization approach for a bank portfolio, which applies different methodologies of risk measurement in the context of actual regulatory requirements. We illustrated by an optimization example the generation of risk-return efficient portfolios with respect to pillar 1 and pillar 2 requirements. We analyzed the effects of different approaches of internal risk aggregation and suggested a systematic approach for risk strategy formulation based on risk-return optimized portfolios under the ICAAP. There is a need for further research and practical implementations especially for integrated risk measurement, risk relations modeling and capital allocation. A future extension of this case study may be based on simulation-based scenarios for the sub-portfolios, especially for the credit portfolios. Different copulae approaches, linking the marginal sub-portfolio distributions to the portfolio loss distribution, may be analyzed in this context. We suggest examining which of the copulae approaches discussed in the literature are the most risk-return efficient in the use of economic capital, that is which copula approach allows to achieve maximum returns in the setting of the integrated optimization problem (Problem 3). Furthermore, the derivation of efficient capital allocation strategies for the optimized portfolios needs to be considered more in depth. Some practical issues of applying the Euler-Allocation principle are not sufficiently investigated, in particular the treatment of capital allocation to assets with negative risk contributions.

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Endnotes

[1] Basel (2004), Part 3: The second pillar – The Supervisory Review Process, § 725.

[2] For a preceding problem formulation refer to Theiler (2004).

[3] See Rosenberg and Schuermann (2004) for a review on realted literature on integrated risk modelling.

[4] Rosenberg and Schuermann (2004), p.10 and Hull (2007), p.375.

[5] Rosenberg and Schuermann (2004), p.3., and Hull (2007), p.375.

[6] The copula couples the marginal distributions together to form a joint distribution. The dependence relationship is entirely determined by the copula, while scaling and shape (mean, standard deviation, skewness, and kurtosis) are entirely determined by the marginals. Rosenberg and Schuermann (2004), p.13.

[7]Hull (2007), p.152.

[8] Conditional Value-at-Risk is a coherent risk measure in the sense of Artzner, Delbaen, Eber, and Heath (1997, 1999), see proof in Rockafellar and Uryasev (2002) and a coherent deviation measure in the sense of Rockafellar and Uryasev (2006b). For a discussion of the appropriateness of CVaR also refer to Acerbi and Tasche (2000), Bertsimas (2004), for a comparison of VaR and CVaR to Yoshiba and Yamai (2001a,b).

[9] See Basel(1996), Introduction II (b), p. 8. For an explanation, refer also to United States Accounting Office (1998), p. 123.

[10] Basel (2004), Part 2 I B 44.

[11] Basel (2004), Part 2 I 40.

[12] The bank book comprises all 'non-trading' assets, while the trading book comprises all positions the bank is holding for short term trading purposes. See, Basel(1996), Introduction I (a), paragraph 2.

[13] Only banks with a *significant* market risk exposure are required to calculate a risk-based capital ratio that takes into account market risk in addition to credit and operational risk.

[14] For definition refer to Basel(1996), Introduction II (a), p. 7. For an explanation, see also United States Accounting Office (1998), pp. 119 and 122.

[15] Basel (1988), part I; for a summary, see, also United States Accounting Office (1998), pp. 116-127.

[16] Basel (1988), part IIa 1.

[17] Basel (2004), Part II, 2.

[18] Basel (2004), Part VI and Basel (1996).

[19] We are using 1 year log returns for both market and credit assets and a 99% confidence interval for VaR Deviation. Refer also to section 3.

[20] This study should further be extended by an inclusion of copulae approaches for integrated risk measurement.

[21] http://www.federalreserve.gov/releases.

[22] Form H.8 (510): Assets and Liabilities of Commercial Banks in the United States of December 2007. We omitted Consumer Credits, as we lacked sufficient data input for the historical index returns.

[23] refer to http://www.federalreserve.gov/releases. data of December 31, 2007, scaled up to the next 10 billions; Largest Banks List: Insured U.S.-chartered Commercial Banks that have consolidated Assets of $300 million or more, ranked by consolditated assets as of December 31, 2007.

[24] We assume that the bank only uses tier 1 and tier 2 capital. We assume all other risks under pillar 1 and pillar 2, which are treated as constants in the optimization model, equal to zero.

[25] In the context of this analysis we abstract from considering further cost components in the nominator of RORAC. For practical implementation the expected return should be adjusted especially for transaction cost, expected and unexpected losses, i. e. capital costs.

[26] Refer to Denault (1999), Tasche (1999).

[27] The difference between Total Capital and VaR Deviation of the portfolios (129.44) is due to approximation error in the derivatives calculation