*Journal of Risk* **20**(4), 75–93
DOI: 10.21314/JOR.2018.383



**Research Paper**

# The CoCVaR approach: systemic risk contribution measurement

**Wei-Qiang Huang[1] and Stan Uryasev[2]**

[1]School of Business Administration, Northeastern University, No 3-11, Wenhua Road, Shenyang City, Liaoning Province, People's Republic of China; email: hwqneu@hotmail.com
[2]Risk Management and Financial Engineering Lab, Department of Industrial and Systems Engineering, University of Florida, PO Box 116595, 303 Weil Hall, Gainesville, FL 32611, USA; email: uryasev@ufl.edu

(Received September 20, 2016; revised December 27, 2016; accepted February 17, 2017)

## ABSTRACT

Systemic risk is the risk that the defaults of one or more institutions trigger a collapse of the entire financial system. In this paper, we propose a measure for systemic risk, CoCVaR, the conditional value-at-risk (CVaR) of the financial system conditional on an institution being in financial distress. This measure is similar to Adrian and Brunnermeier's CoVaR from 2008, but we change the systemic risk from VaR to CVaR. This measure considers severe losses of the financial system beyond VaR. CoCVaR is estimated using CVaR (superquantile) regression. We define the systemic risk contribution of an institution as the difference between CoCVaR conditional on the institution being under distress and the CoCVaR in the median state of the institution. We estimate the systemic risk contributions of the ten largest publicly traded banks in the United States for a sample period February 2000 to January 2015 and compare CoCVaR and CoVaR risk contributions for this period. We find that the new CoCVaR provides a unique perspective on the systemic risk contribution.

**Keywords:** systemic risk contribution; conditional value-at-risk (CVaR); CoCVaR; regression; bank.

Corresponding author: S. Uryasev

Print ISSN 1465-1211 | Online ISSN 1755-2842

## 1 INTRODUCTION

Systemic risk in the financial system is the risk that the failure of an institution to meet its contractual obligations may in turn cause other institutions to default, with the chain reaction leading to broader financial difficulties (Bank for International Settlements 1994). The spread of the failure of one institution to other institutions is implied by financial links between institutions. These financial links include interbank loans, payment systems and over-the-counter derivatives positions (Krause and Giansante 2012). Measuring the contribution of each institution to overall systemic risk can help to identify institutions that make significant contributions to systemic risk. Stricter regulatory requirements for these institutions would stop the tendency to generate systemic risk.

Adrian and Brunnermeier (2008) proposed a measure for systemic risk called CoVaR, namely the value-at-risk (VaR) of the financial system conditional on an institution being under distress. They defined an institution's contribution to systemic risk as the difference between CoVaR conditional on the institution being under distress and CoVaR in the median state. Girardi and Ergun (2013) modified Adrian and Brunnermeier's CoVaR, and changed the definition of financial distress from an institution being exactly at VaR to it exceeding VaR. This change considered more severe distress events that are further in-the-tail. López-Espinosa *et al* (2012) identified the main factors contributing to systemic risk in international large-scale complex banks using the CoVaR approach. They found that short-term wholesale funding is a key factor in triggering systemic risk episodes. Borri *et al* (2014) estimated the systemic risk contribution of Italian-listed banks for the period 2000–11 with the CoVaR methodology. They found that both bank size and leverage are important predictors of systemic risk.

Three ways to calculate CoVaR are considered in the literature:

(1) quantile regression (Adrian and Brunnermeier 2008; López-Espinosa *et al* 2012; Borri *et al* 2014);

(2) multivariate GARCH (generalized autoregressive conditional heteroscedasticity) (Girardi and Ergun 2013); and

(3) the copula method (Reboredo and Ugolini 2015).

In the multivariate GARCH estimation method, the first step estimates the VaR of each financial distribution, and then the bivariate dynamic conditional correlation model is used to estimate the joint distribution of the joint return of the financial system and individual financial institutions. Further, the CoVaR value is calculated by the numerical evaluation of a double integral. For the copula-based approach, the joint distribution of the pair return is expressed by a copula. This method allows the flexible modeling of marginal distributions and the dependence structure. However, both the GARCH and copula methods are too complicated to be widely used by practitioners and regulators, while quantile regression is a relatively simple approach.

The VaR is a quantile of a loss distribution. It is used heavily in various engineering applications, including financial ones. Although VaR is a very popular measure of risk, it has undesirable mathematical characteristics, such as a lack of convexity and discontinuity for discrete distributions. VaR is an "optimistic" measure of risk being a lower bound of the tail. Therefore, it does not measure extreme events. Further, VaR is difficult to optimize when it is calculated with scenarios. The conditional value-at-risk (CVaR) considered by Rockafellar and Uryasev (2000) is similar to the VaR risk measure. CVaR equals the average of some percentage of the worst-case loss scenarios. Therefore, it accounts for extreme events. CVaR has superior mathematical properties to VaR (see the comparative study of VaR and CVaR by Sarykalin *et al* (2008)). CVaR is a so-called coherent risk measure (transition-equivariant, positively homogeneous, convex and monotonic) as studied by Pflug (2000) and Rockafellar and Uryasev (2002).

This paper proposes a new systemic risk measure, CoCVaR, which is inspired by the definition of CoVaR (Adrian and Brunnermeier 2008). We define CoCVaR as the CVaR of the financial system conditional on institutions already being under distress. This allows consideration of severe losses of the financial system in the tail of the return distribution. We define the systemic risk contribution of an institution as the change from its CoCVaR in its benchmark state to its CoCVaR under financial distress. Note that Adrian and Brunnermeier (2016) mentioned that CoVaR can be adapted for other "corisk measures", such as coexpected shortfall (CoES). The CoES definition (the coexpected loss conditional on a VaR event) coincides with our CoCVaR definition. Our paper takes advantage of the recently developed linear regression methodology for the estimation of CVaR (Rockafellar *et al* 2014) and an efficient implementation of this regression in the Portfolio Safeguard (PSG) software package (see American Optimal Decisions: www.aorda.com). We conducted a case study based on weekly data from February 2000 to January 2015 for the ten largest publicly traded banks in the United States. We calculated the banks' time-varying CoVaR and CoCVaR estimates and compared the results.

The remainder of the paper is organized in four sections. Section 2 presents the methodology, including the definition and estimation method for CoVaR and CoCVaR. Section 3 presents the case study. Section 4 concludes.

# 2 METHODOLOGY

## 2.1 CoVaR and CoCVaR definitions

Let $X^{\text{sys}}$ define a random state of a financial system, and let $\boldsymbol{X}$ be a vector of random factors. We suppose that the vector $(X^{\text{sys}}, \boldsymbol{X})$ has some joint probability distribution. CoVaR and CoCVaR are defined as follows.

Definition 2.1 The CoVaR of the system is defined as the VaR with level $\alpha$ of the conditional random variable $X^{\text{sys}} \mid \boldsymbol{X}$, conditional on the event that factors are in a measurable set $\boldsymbol{C}$:

$$\text{CoVaR}_{\alpha}^{\text{sys}} = \text{VaR}_{\alpha}(X^{\text{sys}} \mid \boldsymbol{X} \in \boldsymbol{C}) = \inf\{c \in \mathbb{R} \colon \Pr(X^{\text{sys}} > c \mid \boldsymbol{X} \in \boldsymbol{C}) \leqslant 1 - \alpha\}. \tag{2.1}$$

Definition 2.2 The CoCVaR of the system is defined as the CVaR with level $\alpha$ of the conditional random variable $X^{\text{sys}} \mid \boldsymbol{X}$, conditional on the event that factors are in a measurable set $\boldsymbol{C}$:

$$\text{CoCVaR}_{\alpha}^{\text{sys}} = \text{CVaR}_{\alpha}(X^{\text{sys}} \mid \boldsymbol{X} \in \boldsymbol{C}). \tag{2.2}$$

Theorem 2.3 *CoCVaR can be presented as follows.*

(1) *If $X^{\text{sys}} \mid \boldsymbol{X}$ is a random variable having a continuous distribution, then*

$$\text{CoCVaR}_{\alpha}^{\text{sys}} = \mathbb{E}(X^{\text{sys}} \mid X^{\text{sys}} > \text{CoVaR}_{\alpha}^{\text{sys}},\ \boldsymbol{X} \in \boldsymbol{C}). \tag{2.3}$$

(2) *If $X^{\text{sys}} \mid \boldsymbol{X}$ is a random variable having a general distribution, then*

$$\begin{aligned} \text{CoCVaR}_{\alpha}^{\text{sys}} &= \text{CoVaR}_{\alpha}^{\text{sys}} + \frac{1}{1-\alpha}\mathbb{E}([X^{\text{sys}} - \text{CoVaR}_{\alpha}^{\text{sys}}]^{+} \mid \boldsymbol{X} \in \boldsymbol{C}) \\ &= \min_{c}\left\{c + \frac{1}{1-\alpha}\mathbb{E}([X^{\text{sys}} - c]^{+} \mid \boldsymbol{X} \in \boldsymbol{C})\right\}, \end{aligned} \tag{2.4}$$

*where* $\text{CoVaR}_{\alpha}^{\text{sys}}$ *is a minimizer in (2.4).*

Proof This follows from the general formula for the calculation of CVaR presented in Rockafellar and Uryasev (2002). □

## 2.2 CoVaR and CoCVaR estimation in a dynamic setting

Adrian and Brunnermeier (2008) used quantile regression to estimate CoVaR. Consider the following regression equation:

$$X_t^{\text{sys}} = \beta_0^{\alpha} + \beta_1^{\alpha} X_{t1} + \cdots + \beta_n^{\alpha} X_{tn} + \varepsilon_t, \tag{2.5}$$

where $X_1^{\text{sys}}, X_2^{\text{sys}}, \dots, X_T^{\text{sys}}$ are $T$ observations of a random value $X^{\text{sys}}$, and $X_t = (X_{t1}, X_{t2}, \dots, X_{tn})$ are $T$ corresponding observations of a covariate (random factor) vector $\boldsymbol{X} = (X_1, X_2, \dots, X_n)$, $t = 1, \dots, T$. The estimate of the $\alpha$-quantile of $X^{\text{sys}}$ (ie, the VaR of $X^{\text{sys}}$ with confidence level $\alpha$) under the condition that a realization $(X_{t1}, X_{t2}, \dots, X_{tn})$ is observed is equal to $\hat{\beta}_0^{\alpha} + \sum_{i=1}^{n} X_{ti}\hat{\beta}_i^{\alpha}$, where the vector $\hat{\boldsymbol{\beta}}^{\alpha} = (\hat{\beta}_0^{\alpha}, \hat{\beta}_1^{\alpha}, \dots, \hat{\beta}_n^{\alpha})$ is obtained by solving the quantile regression minimization problem (see Koenker and Basset 1978).

Let $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{+} - \boldsymbol{\varepsilon}^{-}$, where $\boldsymbol{\varepsilon}^{+} = \max\{0, L\}$, $\boldsymbol{\varepsilon}^{-} = \min\{0, L\}$ and $L = X^{\text{sys}} - (\beta_0^{\alpha} + \sum_{i=1}^{n} \beta_i^{\alpha} X_i)$. The rescaled Koenker–Bassett error function is defined as follows (see Rockafellar and Uryasev 2013):

$$\boldsymbol{\varepsilon}_{\alpha}(L) = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\alpha}{1-\alpha}\varepsilon_t^{+} + \varepsilon_t^{-}\right]. \tag{2.6}$$

The vector $\hat{\boldsymbol{\beta}}^{\alpha}$ is estimated by solving the following minimization problem:

$$\hat{\boldsymbol{\beta}}^{\alpha} = \arg\min \boldsymbol{\varepsilon}_{\alpha}(L). \tag{2.7}$$

The estimate of the CoVaR with the quantile regression approach at time $t$ is equal to

$$\text{CoVaR}_{t,\alpha}^{\text{sys}} = \hat{\beta}_0^{\alpha} + \hat{\beta}_1^{\alpha} X_{t1} + \cdots + \hat{\beta}_n^{\alpha} X_{tn}. \tag{2.8}$$

Similar to the quantile regression, the estimate of $\alpha$ − CVaR of $X^{\text{sys}}$ (ie, the CVaR of $X^{\text{sys}}$ with confidence level $\alpha$ under the condition that realization $(X_{t1}, X_{t2}, \dots, X_{tn})$ is observed) is equal to $\hat{\beta}_0^{\prime\alpha} + \sum_{i=1}^{n} X_{ti}\hat{\beta}_i^{\prime\alpha}$. The optimal vector $\hat{\boldsymbol{\beta}}^{\prime\alpha} = (\hat{\beta}_0^{\prime\alpha}, \hat{\beta}_1^{\prime\alpha}, \dots, \hat{\beta}_n^{\prime\alpha})$ is obtained by minimizing the CVaR (superquantile) error, as defined in Rockafellar *et al* (2014). This statement follows from a general quadrangle theory (see Rockafellar and Uryasev 2013). According to the general theory, if some "quadrangle" includes an "error" and a "statistic", then this statistic can be estimated using regression with the error function. Rockafellar *et al* (2014) showed that the CVaR (superquantile) error function is equal to

$$\bar{\varepsilon}_{\alpha}(L) = \frac{1}{1-\alpha}\int_0^1 \text{CVaR}_{\gamma}^{+}(L)\,\mathrm{d}\gamma - \mathbb{E}[L], \tag{2.9}$$

and the CVaR (superquantile) statistic equals

$$\bar{S}(L) = \text{CVaR}_{\alpha}(L), \tag{2.10}$$

where $\alpha$ = confidence level $\in (0, 1)$. Therefore, CoCVaR can be estimated using CVaR (superquantile) regression:

$$\text{CoCVaR}_{t,\alpha}^{\text{sys}} = \hat{\beta}_0^{\prime\alpha} + \hat{\beta}_1^{\prime\alpha} X_{t1} + \cdots + \hat{\beta}_n^{\prime\alpha} X_{tn}. \tag{2.11}$$

## 2.3 Systemic risk contribution

The Dow Jones US Financials Index represents the financials industry as defined by the industry classification benchmark (ICB). It measures the performance of the financial sector of the US equity market, and includes companies in industry groups, such as banks, nonlife insurance, life insurance, real estate and general finance. The index is a subset of the Dow Jones US Index and is capitalization weighted. We denote by $X_t^{\text{sys}}$ a scaled weekly log return of this index:

$$X_t^{\text{sys}} = 100 \ln \frac{I_t}{I_{t-1}}, \tag{2.12}$$

where $I_t$ is the index value at the end of week $t$. Similarly, the $i$th financial institution's scaled weekly log return $R_t^i$ equals

$$R_t^i = 100 \ln \frac{P_t^i}{P_{t-1}^i}, \tag{2.13}$$

where $P_i^t$ is the stock closing price at the end of week $t$.

Let us consider financial institution $i$. We denote the factors for this institution by $\boldsymbol{X} = \{R^i, \boldsymbol{M}\}$, where $R^i$ is the financial institution's weekly log return variable and $\boldsymbol{M}$ represents a vector of common lagged state variables (which will be introduced later on). The $t$th observation of this vector $\boldsymbol{X}$ equals $\{R_t^i, \boldsymbol{M}_{t-1}\}$. Similarly to the method in Adrian and Brunnermeier (2008), the contribution to systemic risk of financial institution $i$ can be measured as the difference between the VaR of $X_t^{\text{sys}}$ conditional on the distress of a particular financial institution $i$ (ie, $R_t^i$ equals its VaR) and the VaR of $X_t^{\text{sys}}$ conditional on the median state of the institution $i$ (ie, $R_t^i$ equals its median). We define financial institution $i$'s contribution to the financial system, $\Delta\text{CoVaR}_{t,\alpha}^{\text{sys}|i}$, by

$$\begin{aligned}
\Delta\text{CoVaR}_{t,\alpha}^{\text{sys}|i} &= \text{CoVaR}_{t,\alpha}^{\text{sys}|R_t^i=\text{VaR}_{t,\alpha}^i} - \text{CoVaR}_{t,\alpha}^{\text{sys}|R_t^i=\text{median}_t^i} \\
&= \text{VaR}_\alpha(X_t^{\text{sys}} \mid \boldsymbol{X} = \{\text{VaR}_{t,\alpha}^i, \boldsymbol{M}_{t-1}\}) \\
&\quad - \text{VaR}_\alpha(X_t^{\text{sys}} \mid \boldsymbol{X} = \{\text{VaR}_{t,0.5}^i, \boldsymbol{M}_{t-1}\}).
\end{aligned} \tag{2.14}$$

Similarly, using CoCVaR, the contribution to systemic risk of financial institution $i$ can also be measured as the difference between the CVaR of $X_t^{\text{sys}}$ conditional on the distress of a particular financial institution $i$ (ie, $R_t^i$ equals its VaR) and the CVaR of $X_t^{\text{sys}}$ conditional on the median state of the institution $i$ (ie, $R_t^i$ equals its median). We denote $i$'s contribution to the financial system, $\Delta\text{CoCVaR}_{t,\alpha}^{\text{sys}|i}$, by

$$\begin{aligned}
\Delta\text{CoCVaR}_{t,\alpha}^{\text{sys}|i} &= \text{CoCVaR}_{t,\alpha}^{\text{sys}|R_t^i=\text{VaR}_{t,\alpha}^i} - \text{CoCVaR}_{t,\alpha}^{\text{sys}|R_t^i=\text{median}_t^i} \\
&= \text{CVaR}_\alpha(X_t^{\text{sys}} \mid \boldsymbol{X} = \{\text{VaR}_{t,\alpha}^i, \boldsymbol{M}_{t-1}\}) \\
&\quad - \text{CVaR}_\alpha(X_t^{\text{sys}} \mid \boldsymbol{X} = \{\text{VaR}_{t,0.5}^i, \boldsymbol{M}_{t-1}\}).
\end{aligned} \tag{2.15}$$

$\mathrm{VaR}^i_{t,\alpha}$ and $\mathrm{VaR}^i_{t,0.5}$ can be estimated using quantile regression:

$$R^i_t = \gamma^i_1 + \gamma^i_2 X_{(t-1)2} + \cdots + \gamma^i_n X_{(t-1)n} + \varepsilon_t. \tag{2.16}$$

## 2.4 State variables

The corisk measures rely on the state variables and firm-specific variables. For example, wholesale funding is the most relevant systemic factor of large international banks (López-Espinosa *et al* 2012); the banks' holding of certain specific types of derivatives, the proportion of nonperforming loans to total loans and the leverage ratio all have an effect on the bank's systemic risk contribution (Mayordomo *et al* 2014). The studies by López-Espinosa *et al* and Mayordomo *et al* investigate the relationships between financial institutions' characteristics and their systemic risk contribution. However, the focus of this paper is to propose a corisk measure CoCVaR and its estimation method, and to compare CoCVaR and CoVaR via a case study. As a result, we do not introduce firm-specific variables to estimate the CoCVaR and $\Delta$CoCVaR. Similarly, in their CoVaR and $\Delta$CoVaR estimations, Adrian and Brunnermeier (2008, 2016) only introduce state variables.

To estimate the time-varying $\mathrm{CoVaR}_t$, $\mathrm{CoCVaR}_t$ and $\mathrm{VaR}_t$, we include the set of state variables $\boldsymbol{M}_t$ considered in Adrian and Brunnermeier (2008). These state variables are the following.

(1) The Chicago Board Options Exchange (CBOE) Volatility Index (VIX), which captures the implied volatility in the stock market reported by the CBOE.

(2) A short-term "liquidity spread", defined as the difference between the three-month repurchase agreement (repo) rate and the three-month Treasury bill (T-bill) rate. This liquidity spread measures short-term liquidity risk. We obtain the three-month T-bill rate from the Federal Reserve Bank's H.15 report.[1] We use the three-month general collateral repo rate from Bloomberg.

(3) The change in the three-month T-bill rate: Adrian and Brunnermeier (2008) found that the change in the three-month T-bill rate is most significant in explaining the tails in asset returns of financial institutions.

(4) The change in the slope of the yield curve, measured by the yield spread between the ten-year Treasury rate and the three-month bill rate obtained from Federal Reserve Bank's H.15 report.

(5) The change in the credit spread between Baa-rated bonds and the Treasury rate (with the same ten-year maturity) from the Federal Reserve Bank H.15 report.

[1] See www.federalreserve.gov/releases/h15/.

(6) The weekly equity market return: we use the Standard & Poor's 500 Index to calculate the equity market return downloaded from Yahoo! Finance.

(7) The weekly real estate sector return in excess of the market return: we use the Dow Jones US Real Estate Index downloaded from Google Finance to calculate the real estate sector return.

## 3 CASE STUDY

### 3.1 Numerical implementation

We used Portfolio Safeguard (PSG) for numerical calculations. PSG has the pre-coded Koenker–Bassett and CVaR (superquantile) errors; therefore, quantile and CVaR regression problems with these functions can be easily coded. Moreover, additional constraints can be imposed, such as a cardinality constraint on the number of regression variables. The case study data and numerical runs were posted to the second author's research web page (http://bit.ly/2FCAPBq).

In particular, we posted the PSG codes solving the quantile regression and CVaR regression in the `Run-File` and MATLAB environments. In addition, we posted the MATLAB code that calculates $\Delta$CoVaR and $\Delta$CoCVaR for the ten largest US banks. PSG codes are very efficient and designed to solve large-scale problems with millions of observations.

### 3.2 Financial institutions

We considered the ten largest (by total assets) publicly traded banks in the United States as of December 31, 2014:

(1) JPMorgan Chase & Company (JPM);

(2) Bank of America (BAC);

(3) Citigroup Inc (C);

(4) Wells Fargo & Company (WFC);

(5) The Bank of New York Mellon Corporation (BK);

(6) US Bancorp (USB);

(7) Capital One Financial Corporation (COF);

(8) PNC Financial Services Group Inc (PNC);

(9) State Street Corporation (STT);

(10) The BB&T Corporation (BBT).

**TABLE 1** State variable summary statistics.

| Variable | Mean | SD | Min | Max | 1% stress level |
|---|---|---|---|---|---|
| VIX | 20.7756 | 9.1372 | 10.0200 | 79.1300 | 51.8771 |
| Liquidity spread (%) | 0.1855 | 0.2006 | –0.1300 | 1.3050 | 0.2735 |
| 3M Treasury change (%) | –0.0022 | 0.1762 | –0.9900 | 3.9000 | –0.2014 |
| Term spread change (%) | 1.9965 | 1.1799 | –0.6500 | 3.8200 | 2.8100 |
| Credit spread change (%) | 2.7000 | 0.8103 | 1.4800 | 6.1000 | 5.0386 |
| Equity return (%) | 0.0521 | 2.6222 | –20.0838 | 11.3559 | –9.2436 |
| Real estate excess return (%) | 0.0748 | 2.6300 | –17.5340 | 12.3683 | –0.4582 |

All data is weekly. SD denotes standard deviation.

We considered the period from February 18, 2000 to January 30, 2015 (754 weeks). The data thus covers two recessions (2001 and 2007–9) and two financial crises (2000 and 2008). We downloaded the weekly Dow Jones US Financials Index and financial institutions' closing prices from Yahoo! Finance.

## 3.3 Calculation results

### *3.3.1 State variable statistics*

Table 1 summarizes statistics for the state variables. The 1% stress level is the average value for the observations of each respective state variable during the 1% worst weeks for financial system returns. For example, the average of VIX during the stress periods is 51.8771, as the worst times for the financial system include those when the VIX was highest. Similarly, the stress level corresponds to a high level of liquidity spread, a sharp increase in the term and credit spreads and large negative equity return realizations.

### *3.3.2 VaR of financial institutions*

First, we use (2.16) to obtain time-dependent VaR by running quantile regressions of the banks' stock log returns on the lagged state variables. In the following analysis, we consider the 10% quantiles, corresponding to the worst seventy-five weeks over the sample horizon. Table 2 shows the regression coefficients for different banks. We denote the state variables by VIX, LS, TC, TSC, CSC, ER and REER, corresponding to VIX, liquidity spread, three-month Treasury change, term spread change, credit spread change, equity market return and real estate sector excess return. According to (2.14) and (2.15), we also obtain a time variation of VaR by running quantile regressions of the banks' stock log returns on the lagged state variables with a 50%

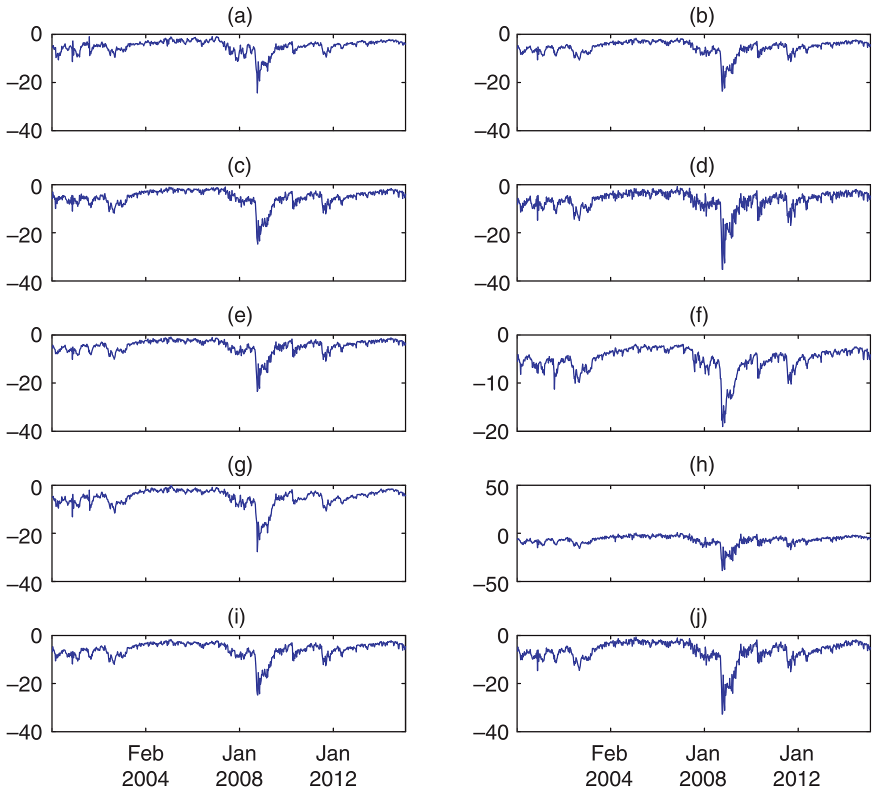
**TABLE 2** State variable coefficients of 10% quantile regression for calculating VaR.

| Bank | Intercept | VIX | LS | TC | TSC | CSC | ER | REER | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| JPM | 2.9665 | –0.2365 | –2.8862 | –4.4554 | 0.3508 | –1.5738 | 0.1100 | 0.0418 | 0.1653 |
| BAC | 5.5247 | –0.2401 | –1.9837 | –7.1799 | 0.6032 | –2.9479 | 0.1977 | 0.2091 | 0.1512 |
| C | 7.1515 | –0.2412 | –5.7393 | –7.4349 | 0.1573 | –3.1743 | 0.3411 | 0.4758 | 0.1669 |
| WFC | 5.4068 | –0.1821 | –3.6496 | –7.1027 | 0.4596 | –2.6132 | –0.0949 | 0.1305 | 0.1749 |
| BK | 1.5293 | –0.2108 | –0.2371 | 1.0039 | 0.0945 | –0.8835 | –0.0176 | –0.0409 | 0.1513 |
| USB | 3.0452 | –0.2276 | –2.9354 | –4.3543 | 0.2581 | –1.1180 | 0.0960 | 0.1482 | 0.1671 |
| COF | 3.2689 | –0.3557 | –2.1423 | –8.8168 | 0.1454 | –0.9198 | 0.2784 | 0.2684 | 0.1864 |
| PNC | 3.9091 | –0.2220 | –0.5613 | –5.4348 | 0.3139 | –1.9098 | 0.1645 | –0.0012 | 0.1680 |
| STT | 2.7207 | –0.2100 | –2.8971 | –4.5881 | 0.0993 | –1.3069 | 0.0880 | 0.1125 | 0.1118 |
| BBT | 3.3468 | –0.1883 | –6.9436 | –6.7503 | –0.3133 | –0.9476 | –0.0818 | 0.1504 | 0.1610 |

**TABLE 3** State variable coefficients of 50%-quantile regression for calculating VaR.

| Bank | Intercept | VIX | LS | TC | TSC | CSC | ER | REER | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| JPM | 1.3272 | 0.0164 | −2.4076 | −1.4877 | −0.2589 | −0.2658 | 0.1057 | 0.0244 | 0.0167 |
| BAC | 1.4902 | −0.0114 | −1.7703 | −2.7344 | −0.0509 | −0.3315 | −0.1713 | −0.0005 | 0.0137 |
| C | 1.6323 | 0.0592 | −2.5601 | −1.2293 | −0.2666 | −0.7022 | −0.1148 | −0.0275 | 0.0155 |
| WFC | 0.2078 | 0.0094 | −0.7831 | −1.4109 | −0.1576 | 0.1164 | −0.2213 | 0.0061 | 0.0244 |
| BK | 0.4681 | −0.0112 | −1.3392 | −1.8185 | −0.2519 | 0.1696 | −0.1177 | −0.1516 | 0.1042 |
| USB | 0.0483 | −0.0339 | −1.3643 | −3.1755 | −0.1211 | 0.5013 | −0.1343 | −0.0597 | 0.0183 |
| COF | 1.9304 | −0.0272 | −2.2011 | −3.0571 | 0.0086 | −0.3338 | −0.2330 | −0.0006 | 0.0204 |
| PNC | 0.9703 | 0.0311 | −1.9922 | −1.6763 | −0.2850 | −0.1871 | −0.1200 | −0.0990 | 0.0167 |
| STT | 0.1098 | −0.0055 | −0.5993 | −1.1773 | −0.1916 | 0.2687 | −0.0945 | −0.1323 | 0.0146 |
| BBT | 0.5151 | −0.0140 | −0.7462 | −0.8163 | −0.0953 | 0.1255 | −0.2455 | 0.0637 | 0.0201 |

**FIGURE 1** The time-varying VaR(0.1) of the banks.




(a) BBT. (b) STT. (c) PNC. (d) COF. (e) USB. (f) BK. (g) WFC. (h) C. (i) JPM. (j) BAC. The VaR(0.1) of banks is estimated using the quantile regression coefficients shown in Table 2.

confidence level. Table 3 gives the regression coefficients for different banks. Figure 1 shows the time-varying VaR(0.1) for each bank.

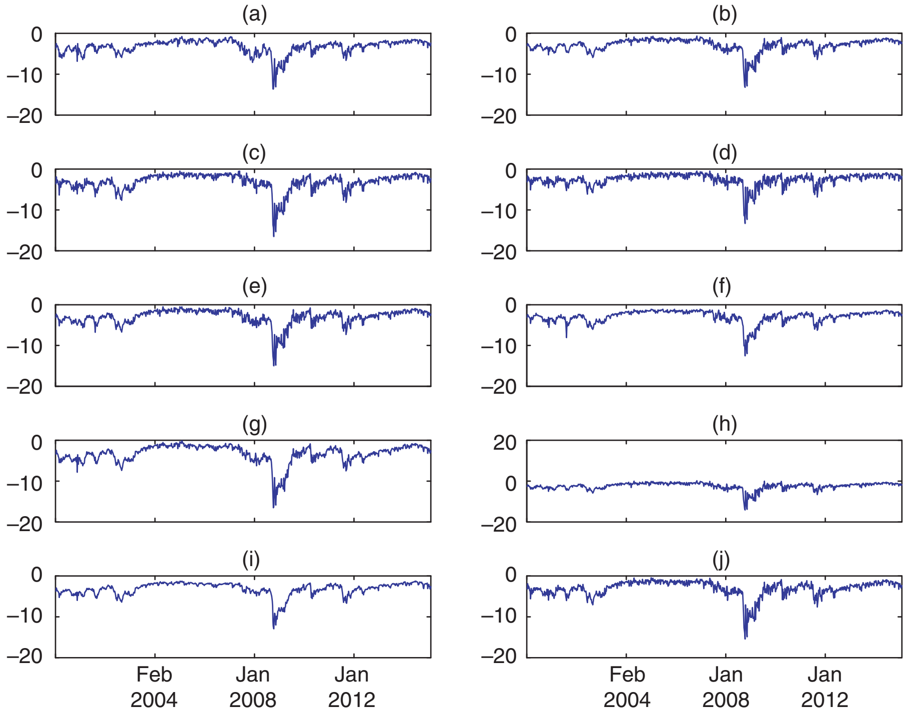
### *3.3.3* CoVaR *and* ΔCoVaR

With quantile regressions (see (2.8)), we obtain the time-dependent 10% VaR of Dow Jones US Financial Index returns for the lagged state variables and the banks' stock returns. Table 4 shows the regression results, where variable "RB" denotes the bank's stock return. It is found that the effects of state variables on the CoVaR have the same sign but different sensitivities for most banks. However, there are some banks that load on some of the state variables in a different direction to most of the other

**TABLE 4** Variable coefficients of 10% quantile regression for estimating CoVaR.

| Bank | Intercept | RB | VIX | LS | TC | TSC | CSC | ER | REER | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| JPM | 1.5946 | 0.5351 | −0.0611 | −3.1603 | −1.4265 | 0.0925 | −0.8000 | 0.0035 | 0.0382 | 0.5151 |
| BAC | 0.9843 | 0.4820 | −0.0723 | −2.6402 | −0.5524 | −0.0266 | −0.4486 | −0.0788 | 0.0078 | 0.5443 |
| C | 1.2434 | 0.3531 | −0.1274 | −2.2025 | −2.7461 | −0.0684 | −0.1341 | −0.0613 | −0.0129 | 0.3709 |
| WFC | 0.5623 | 0.6225 | −0.0826 | −1.1082 | −1.4036 | −0.0433 | −0.3343 | 0.0998 | 0.0567 | 0.5254 |
| BK | 1.3407 | 0.5827 | −0.0331 | −3.9988 | −0.9339 | 0.0208 | −0.9502 | 0.1474 | 0.2076 | 0.4620 |
| USB | 1.5407 | 0.6177 | −0.0743 | −4.7790 | −2.3598 | −0.1643 | −0.5028 | 0.1108 | 0.0794 | 0.4807 |
| COF | 0.4298 | 0.3839 | −0.1066 | −2.5658 | −1.6226 | −0.1343 | −0.1054 | 0.1187 | 0.0921 | 0.4608 |
| PNC | 0.8113 | 0.5833 | −0.1438 | −1.4012 | −1.3111 | 0.0480 | −0.0583 | −0.0181 | 0.0745 | 0.4791 |
| STT | 1.9825 | 0.5119 | −0.0897 | −3.2388 | −3.0902 | 0.0845 | −0.8764 | 0.0416 | 0.1381 | 0.4500 |
| BBT | 1.8500 | 0.6382 | −0.0697 | −2.3886 | 0.4619 | −0.2004 | −0.7857 | 0.1174 | 0.0466 | 0.4970 |

**FIGURE 2** The ΔCoVaR time series for the banks.




(a) BBT. (b) STT. (c) PNC. (d) COF. (e) USB. (f) BK. (g) WFC. (h) C. (i) JPM. (j) BAC. ΔCoVaR is the difference between the CoVaR conditional on the distress of an institution and the CoVaR conditional on the median state of that institution.

banks. For example, the REER has a negative effect on the CoVaR of bank C, which is different from those of the other banks.

We estimated 10% VaR and 50% VaR of each bank's stock return with quantile regression (see (2.8)). Further, these estimates were used in (2.14) to obtain the CoVaR of the financial system. Figure 2 shows the ΔCoVaR time series for each bank. According to Table 4, the CoVaRs of most banks have similar responses to the state variables. As a result, the trends in the ΔCoVaR time series of the different banks are similar.

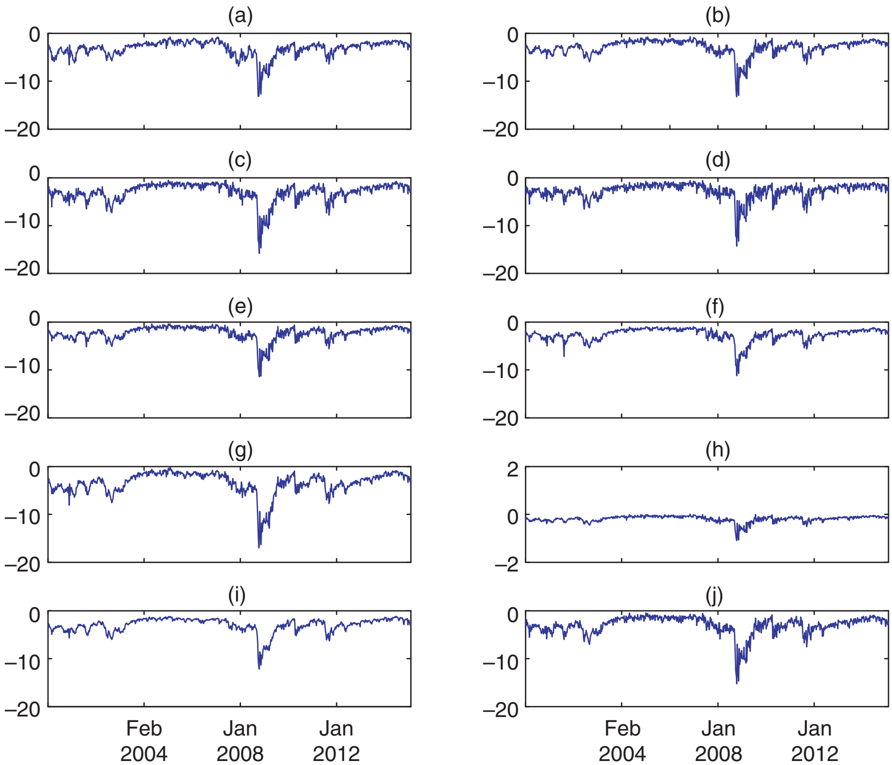
### *3.3.4* CoCVaR *and* ΔCoCVaR

We used CVaR (superquantile) regressions for estimating the time-dependent 10% CVaR of the Dow Jones US Financial Index return on the lagged state variables and

**TABLE 5** Variable coefficients of 10% CVaR (superquantile) regression for estimating CoCVaR.

| Bank | Intercept | RB | VIX | LS | TC | TSC | CSC | ER | REER | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| JPM | 2.1852 | 0.5028 | −0.1500 | −5.7163 | 0.0615 | 0.0184 | −0.6354 | 0.1120 | 0.1199 | 0.5844 |
| BAC | 1.0090 | 0.4752 | −0.0874 | −3.3365 | −0.7511 | −0.0856 | −0.5777 | −0.0218 | 0.0302 | 0.6392 |
| C | 3.2285 | 0.0271 | −0.2148 | −5.2906 | −2.4967 | −0.0376 | −1.2534 | 0.2379 | 0.1459 | 0.3826 |
| WFC | −0.0320 | 0.6388 | −0.1340 | 0.5474 | 2.4059 | 0.3030 | −0.5040 | 0.1767 | 0.0824 | 0.6105 |
| BK | 1.8221 | 0.5212 | −0.1113 | −7.7675 | −3.3682 | −0.2840 | −0.5514 | 0.2170 | 0.2042 | 0.5722 |
| USB | 2.3427 | 0.4697 | −0.1911 | −4.8071 | −2.0186 | 0.0323 | −0.5976 | 0.1442 | 0.1152 | 0.5612 |
| COF | −0.2978 | 0.4114 | −0.1972 | −1.8540 | 2.0546 | −0.1665 | 0.3912 | 0.0575 | 0.2337 | 0.5593 |
| PNC | 0.3212 | 0.5573 | −0.1593 | −4.2826 | 2.7238 | 0.0546 | −0.0980 | 0.1404 | 0.2561 | 0.5403 |
| STT | 2.0689 | 0.5139 | −0.1568 | −7.7920 | −4.5835 | −0.2638 | −0.3464 | 0.1210 | 0.1048 | 0.5300 |
| BBT | 0.4971 | 0.6157 | −0.1749 | 0.0362 | 2.9436 | −0.0838 | −0.1479 | 0.2952 | 0.0366 | 0.5734 |

**FIGURE 3** The ΔCoCVaR time series of the banks.




(a) BBT. (b) STT. (c) PNC. (d) COF. (e) USB. (f) BK. (g) WFC. (h) C. (i) JPM. (j) BAC. ΔCoCVaR is the difference between the CoCVaR conditional on the distress of an institution and the CoCVaR conditional on the median state of that institution.

banks' stock returns. Table 5 presents the regression results. We found that the effects of state variables on the CoCVaR have the same sign but different sensitivities for most banks. However, there are some banks that load on some of the state variables in a different direction to most of the other banks. For example, the ER has a negative effect on the CoCVaR of bank BAC, which is different from those of the other banks.

We estimated the 10% VaR and 50% VaR of each bank's stock return with quantile regression (see (2.8)). Further, these estimates were used in (2.15) to obtain the CoCVaR of the financial system. Figure 3 shows the ΔCoCVaR time series for each bank. According to Table 5, the CoCVaRs of most of the banks have similar responses to the state variables. As a result, the trends in the ΔCoCVaR time series of the different banks are similar.

**TABLE 6** Average VaR, average ΔCoVaR, average ΔCoCVaR and bank rank.

| | Bank | | | | |
|---|---|---|---|---|---|
| | **BBT** | **STT** | **PNC** | **COF** | **USB** |
| Average VaR | −4.9818 (9) | −5.4631 (5) | −5.2872 (6) | −6.6118 (2) | −4.6835 (10) |
| Average ΔCoVaR | −3.3262 (2) | −2.9095 (8) | −3.1842 (4) | −2.5701 (9) | −3.0251 (6) |
| Average ΔCoCVaR | −3.2091 (2) | −2.9210 (6) | −3.0424 (4) | −2.7544 (7) | −2.3004 (9) |

| | Bank | | | | |
|---|---|---|---|---|---|
| | **BK** | **WFC** | **C** | **JPM** | **BAC** |
| Average VaR | −5.1025 (8) | −5.1355 (7) | −7.0717 (1) | −5.9883 (4) | −6.5074 (3) |
| Average ΔCoVaR | −2.9371 (7) | −3.3567 (1) | −2.4824 (10) | −3.2073 (3) | −3.1069 (5) |
| Average ΔCoCVaR | −2.6271 (8) | −3.4444 (1) | −0.1908 (10) | −3.0630 (5) | −3.0135 (3) |

Values stated are the arithmetic mean of weekly time series for each bank. We rank (in parentheses) each bank in order of decreasing absolute values.

### *3.3.5 Comparisons*

Table 6 gives the average VaR, average ΔCoVaR and average ΔCoCVaR at the 10% confidence level during the sample period for each bank, and their respective rankings. The ranking of banks by individual risks measured by VaR and systemic risk contribution measured by ΔCoVaR may not coincide. For example, Citigroup Inc (C) ranks first according to its VaR, but tenth according to its ΔCoVaR. So, a single bank's risk measure does not necessarily reflect the systemic risk. We also find that CoVaR and CoCVaR provide different rankings of systemic risk contributions for some banks (JPM, BAC, BK, USB, COF, STT). For instance, the US Bancorp (USB) ranks sixth according to its average ΔCoVaR, but ninth according to its average ΔCoCVaR. There are two reasons for this difference. First, CoVaR and CoCVaR have different properties and provide different information. The CoVaR measure provides only a lower bound for a financial system's conditional losses in the tail. Meanwhile, the CoCVaR measure accounts for the extent of conditional losses that could be suffered beyond the specified threshold. The conditional losses in the tail could be only slightly higher than the threshold or very far beyond the threshold. The CoCVaR measure may be larger than, smaller than or equal to the CoVaR measure for a bank, depending on the conditional tail distribution of the financial system. Second, the differences between CoVaR and CoCVaR would be larger for some banks and smaller for others. Therefore, these two measures may provide different rankings.

Although $\Delta$CoCVaR and $\Delta$CoVaR provide different rankings for some banks, they provide the same rankings for others. For instance, for the Wells Fargo (WFC), BB&T Corporation (BBT), PNC Financial Services Group Inc (PNC) and Citigroup Inc (C), both the average $\Delta$CoVaR and the average $\Delta$CoCVaR provide the same ranking of their systemic risk contributions.

## 4 CONCLUSION

In this paper, we proposed a new systemic risk measure, CoCVaR, inspired by the definition of CoVaR in Adrian and Brunnermeier (2008). This new definition considers severe losses of the financial system in the tail of the return distribution beyond VaR. CoCVaR measures were estimated using CVaR (superquantile) regression. We proposed a new systemic risk contribution measurement, $\Delta$CoCVaR, which focuses on heavy tails and complements $\Delta$CoVaR. We conducted a case study with US-listed banks and compared VaR, $\Delta$CoVaR and $\Delta$CoCVaR numerically.

We found that banks' rankings according to individual risk measured by VaR and systemic risk contribution measured by $\Delta$CoVaR may not coincide. A bank may have a high systemic risk contribution even though its individual risk may be low. In addition, a single bank's high VaR risk does not necessarily reflect a high systemic risk.

We also found that the $\Delta$CoVaR and $\Delta$CoCVaR measures may give different risk rankings for the same bank. The new CoCVaR focuses on the tail of the distribution and provides a unique perspective on the systemic risk contribution.

## DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

## ACKNOWLEDGEMENTS

We acknowledge the financial support of the National Natural Science Foundation of China (grants 71771042 and 71371044).

## REFERENCES

Adrian, T., and Brunnermeier, M. (2008). CoVaR. Staff Report 348, Federal Reserve Bank of New York.

Adrian, T., and Brunnermeier, M. (2016). CoVaR. *American Economic Review* **106**(7), 1705–1741 (https://doi.org/10.1257/aer.20120555).

Bank for International Settlements (1994). *64th Annual Report*. BIS, Basel.

Borri, N., Caccavaio, M., Giorgio, G. D., and Sorrentino, A. M. (2014). Systemic risk in the Italian banking industry. *Economic Notes* **43**(43), 21–38 (https://doi.org/10.1111/ecno.12015).

Girardi, G., and Ergun, A. T. (2013). Systemic risk measurement: multivariate GARCH estimation of CoVaR. *Journal of Banking and Finance* **37**(8), 3169–3180 (https://doi.org/10.1016/j.jbankfin.2013.02.027).

Koenker, R., and Bassett, G., Jr. (1978). Regression quantile. *Econometrica* **46**(1), 33–50 (https://doi.org/10.2307/1913643).

Krause, A., and Giansante, S. (2012). Interbank lending and the spread of bank failures: a network model of systemic risk. *Journal of Economic Behavior and Organization* **83**(3), 583–608 (https://doi.org/10.1016/j.jbankfin.2013.02.027).

López-Espinosa, G., Moreno, A., Rubia, A., and Valderrama, L. (2012). Short-term wholesale funding and systemic risk: a global CoVaR approach. *Journal of Banking and Finance* **36**(12), 3150–3162 (https://doi.org/10.1016/j.jbankfin.2012.04.020).

Mayordomo, S., Rodriguez-Moreno, M., and Peña, J. I. (2014). Derivatives holdings and systemic risk in the US banking sector. *Journal of Banking and Finance* **45**, 84–104 (https://doi.org/10.1016/j.jbankfin.2014.03.037).

Pflug, G. (2000). Some remarks on the value-at-risk and the conditional value-at-risk. In *Probabilistic Constrained Optimization: Methodology and Applications*, Uryasev, S. (ed). Kluwer Academic, Dordrecht (https://doi.org/10.1007/978-1-4757-3150-7).

Reboredo, J. C., and Ugolini, A. (2015). Systemic risk in European sovereign debt markets: a CoVaR-copula approach. *Journal of International Money and Finance* **51**, 214–244 (https://doi.org/10.1016/j.jimonfin.2014.12.002).

Rockafellar, R. T., and Uryasev, S. (2000). Optimization of conditional value-at-risk. *The Journal of Risk* **2**(3), 21–42 (https://doi.org/10.21314/JOR.2000.038).

Rockafellar, R. T., and Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance* **26**(7), 1443–1471 (https://doi.org/10.1016/S0378-4266(02)00271-6).

Rockafellar, R. T., and Uryasev, S. (2013). The fundamental risk quadrangle in risk management, optimization, and statistical estimation. *Surveys in Operations Research and Management Science* **18**(1), 33–53 (https://doi.org/10.1016/j.sorms.2013.03.001).

Rockafellar, R. T., Royset, J. O., and Miranda, S. I. (2014). Superquantile regression with applications to buffered reliability, uncertainty quantification and conditional value-at-risk. *European Journal of Operations Research* **234**(1), 140–154 (https://doi.org/10.1016/j.ejor.2013.10.046).

Sarykalin, S., Serraino, G., and Uryasev, S. (2008). Value-at-risk vs conditional value-at-risk in risk management and optimization. *Tutorials in Operations Research* **10**, 270–294 (https://doi.org/10.1287/educ.1080.0052).