CASE STUDY: Mortgage Pipeline Hedging (st_dev, meanabs_dev, cvar_dev, var_risk, var_risk_g)

Background

This case study finds optimal pipeline hedging strategies by minimizing four deviation measures: Standard Deviation, Mean Absolute Deviation, CVaR Deviation, Two-tailed VaR[1] deviation. We test in-sample and out-of-sample performance of strategies.

Mortgage lenders usually originate mortgages by selling them in the secondary market. Alternatively, the funding can be obtained by mortgage lenders through securitizing the mortgages in exchange for mortgage backed securities (MBS) and then selling MBS to investors in the secondary market. This case study considers hedging the risk in the mortgage underwriting process known as "pipeline". Mortgage lenders commit to a mortgage interest rate while the loan is in process, typically for a period of 30-60 days. If the rate rises before the loan goes to closing, the value of the loan declines and the lender sells the loan at a lower price. The risk that mortgages in process will fall in value prior to their sale is known as mortgage pipeline risk. Lenders often hedge this exposure by selling forward their expected closing volume or by shorting U.S. Treasury notes, or futures contracts. Fallout refers to the percentage of loan commitments that do not go to closing. The mortgage pipeline risk is affected by fallout. As interest rates fall, the fallout rises because borrowers locked in a mortgage rate are more likely to find better rates with another lender. Conversely, as rates rise the percentage of loans that close increases. The fallout affects the required size of the hedging instrument because it affects the size of the pipeline position to be hedged. At lower rates, fewer rate loans will close and a smaller position in the hedging instrument is needed. Lenders often use options on U.S. Treasury note futures to hedge against the fallout risk (Cusatis and Thomas, 2005).

This case study considers three hedging instruments for hedging the pipeline risk: 5% MBS forward, 5.5% MBS forward, and call options on 10-year Treasury note futures. We ignore transaction costs and allow short sales.

We considered standard deviation, mean absolute deviation, CVaR90 deviation, two-tailed VaR75 and two-tailed VaR90 as the objective function in the minimum risk hedging model. We considered the case with and without the constraint that the average loss is equal to zero.

We paid a special attention to the two-tailed VaR75 and VaR90 minimization problems. Both two-tailed VaR75 and VaR90 problems, with and without constraint on average loss, with time limit 360 and 3600 seconds were solved with: 1) solver VAN, which uses PSG heuristics; 2) solver CARGRB (which calls GUROBI) as a subsolver. The Two-Tail VaR Deviation is not a convex function. CARGRB solver generates Mixed Integer Linear problem (MILP) with additional linear constraints and Boolean variables. CARGRB uses GUROBI as subsolver to solve MILP problem.

For the out-of-sample testing, we partition the 1,000 scenarios into 10 groups with 100 scenarios in each group. For every optimization problem we selected one group for the out-of-sample testing and calculated optimal hedging positions based on the remaining 9 groups containing 900 scenarios. For each group of 100 scenarios we calculated the ex-ante losses (i.e., underperformances of hedging portfolio versus target) with the optimal hedging positions obtained from the 900 scenarios. To estimate the out-of-sample performance, we aggregated the out-of-sample losses from the 10 runs to obtain a combined set including 1,000 out-of-sample losses. Then, we evaluated deviation measures on the out-of-sample 1,000 losses. Also, we calculated in-sample performance by averaging characteristics over in-sample datasets.

In addition, we calculated in-sample and out-of-sample three downside risk measures: 90%-CVaR, 90%-VaR, and Max Loss. For the case without the constraint on the average loss, by minimizing the Two-Tail 90%-VaR Deviation, we obtained the best values for all three considered downside risk measures. Minimization of CVaR deviation leads to good results, whereas minimization of standard deviation gives the worst outcomes for the three downside risk measures. Numerical runs demonstrated that the constraint on the average loss deteriorated both in-sample and out-of-sample results, compared to the case without the constraint.

The case study results were partially reported in (Sarykalin, Serraino, Uryasev, 2008).

[1] For instance Two-Tailed VaR75 is defined as VaR75(L(θ,X))+VaR75(-L(θ,X)), i.e., 75% percentile of loss distribution plus 75% percentile of profit distribution.

References

- Sarykalin, S., Serraino, G., and S . Uryasev (2008): VaR vs CVaR in Risk Management and Optimization. Tutorials in Operations Research, INFORMS.
- Cusatis, P.J. and M.R. Thomas (2005): Hedging Instruments and Risk Management, McGraw-Hill
- Taglia, P. (2003): Risk Management Case Study: How Mortgage Lenders Use Futures to Hedge Pipeline Risk. *Futures Industry Magazine*, September/October.
- Lederman, J. (1997): Handbook of Secondary Marketing, Mortgage Bankers Association of America.
- Hakim, S., Rashidian, M., and E. Rosenblatt (1999): Measuring the Fallout Risk in the Mortgage Pipeline. *The Journal of Fixed Income*, 9, 62–75.

Notations

I= number of hedging instruments; i={1,…,I} index of hedging instrument in the portfolio;

J=number of observations (scenarios);

$\vec{x} = (x_1, x_2, ..., x_I)$ decision vector defining positions in hedging instruments;

θ_{ij} = rate of return of the i-th instrument under scenario j;

$\vec{\theta} = (\theta_0, \theta_1, \theta_2, .., \theta_I)$ = random vector of returns of hedging instruments and benchmark;

$\overrightarrow{\theta_j} = (\theta_{0j}, \theta_{1j}, .., \theta_{Ij})$ = vector of returns of instruments, $i=1,...,I$, under scenarios j;

$L(\vec{x}, \overrightarrow{\theta_j}) = \theta_{0j} - \sum_{i=1}^{I} \theta_{ij} x_i$ = loss under scenario j;

$Avg(L(\vec{x}, \vec{\theta})) = \frac{1}{J} \sum_{j=1}^{J} L(\vec{x}, \vec{\theta}_j) =$ average of loss;

$CVaR_\alpha^\Delta(L(\vec{x}, \vec{\theta})) =$ α%-CVaR Deviation of loss;

$MAD(L(\vec{x}, \vec{\theta})) =$ Mean absolute deviation of loss;

$\sigma(L(\vec{x}, \vec{\theta})) =$ Standard deviation of loss;

$VaR_\alpha^\Delta(L(\vec{x}, \vec{\theta})) =$ α%-VaR Deviation of loss;

$TwoTailVaR_\alpha^\Delta(L(\vec{x}, \vec{\theta})) = VaR_\alpha(L(\vec{x}, \vec{\theta})) + VaR_\alpha(-L(\vec{x}, \vec{\theta})) =$ Two Tail α%-VaR Deviation of loss.

Optimization Problems

Problem 1. Standard Deviation minimization. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Standard Deviation

$$\min_x \sigma(L(\vec{x}, \vec{\theta}))$$

Problem 1a. Standard Deviation minimization with constraint on the average loss. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Standard Deviation

$$\min_x \sigma(L(\vec{x}, \vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 2. Mean Absolute Deviation minimization. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Mean Absolute Deviation

$$\min_x MAD(L(\vec{x},\vec{\theta}))$$

Problem 2a. Mean Absolute Deviation minimization with constraint on the average loss. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Mean Absolute Deviation

$$\min_x MAD(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 3. CVaR Deviation minimization. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing 90%-CVaR Deviation

$$\min_x CVaR_{0.9}^{\Delta}(L(\vec{x},\vec{\theta}))$$

Problem 3a. CVaR Deviation minimization with constraint on the average loss. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing 90%-CVaR Deviation

$$\min_x CVaR_{0.9}^{\Delta}(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 4. Two-Tailed VaR75 Deviation minimization. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Two-Tail 75%-VaR Deviation

$$\min_x TwoTailVaR_{0.75}^{\Delta}(L(\vec{x},\vec{\theta}))$$

Problem 4a. Two-Tailed VaR75 Deviation minimization with constraint on the average loss. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Two-Tail 75%-VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.75}(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 4b. Two-Tailed VaR90 Deviation minimization. 10-fold Cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Two-Tail 90%-VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.90}(L(\vec{x},\vec{\theta}))$$

Problem 4c. Two-Tailed VaR90 Deviation minimization with constraint on the average loss. 10-fold cross-validation (10 in-sample and 10 out-of-sample datasets).

Minimizing Two-Tail 90%-VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.90}(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 5. Two-Tailed VaR75 minimization (MILP formulation)

Minimizing Two-Tail 75%- VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.75}(L(\vec{x},\vec{\theta}))\,.$$

Problem 5a. Two-Tailed VaR75 minimization with constraint on the average loss (MILP formulation)

Minimizing Two-Tail 75%-VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.75}(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$

Problem 5b. Two-Tailed VaR90 minimization (MILP formulation)

Minimizing Two-Tail 90%-VaR Deviation

$$\min_x TwoTailVaR^{\Delta}_{0.90}(L(\vec{x},\vec{\theta}))\,.$$

Problem 5 c. Two-Tailed VaR90 minimization with constraint on the average loss (MILP formulation)

Minimizing Two-Tail 90%-VaR Deviation

$$\min_{x} TwoTailVaR^{\Delta}_{0.90}(L(\vec{x},\vec{\theta}))$$

subject to

average loss constraint

$$Avg(L(\vec{x},\vec{\theta})) = 0\,.$$