Buffered Probability of Exceedance (bPOE) Ratings for Synthetic Instruments

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Outline

1. Background
2. Problem with ratings
3. VaR and CVaR
4. bPOE ratings methodology

Bond Ratings

- Most bonds have rating assigned by one of rating agencies (big three: S&P 500, Moody's, Fitch)
- Ratings indicate how risky is the bond (how likely it will default)
- Historical data confirm that ratings are good predictors of Probability of Default (PD)

S&P default probabilities (USA companies) for each rating

Average Cumulative Default Rates For Corporates By Region (1981-2015) (%)

	--Time horizon (years)--														
Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
U.S.															
AAA	0.00	0.04	0.17	0.29	0.42	0.54	0.59	0.67	0.76	0.86	0.90	0.95	1.00	1.10	1.21
AA	0.04	0.08	0.18	0.32	0.46	0.61	0.76	0.88	0.98	1.09	1.19	1.28	1.37	1.45	1.55
A	0.08	0.21	0.37	0.56	0.75	0.97	1.22	1.45	1.70	1.95	2.18	2.38	2.58	2.75	2.95
BBB	0.23	0.61	1.02	1.54	2.10	2.65	3.15	3.65	4.15	4.64	5.12	5.50	5.86	6.23	6.60
BB	0.81	2.51	4.58	6.60	8.38	10.14	11.61	12.96	14.17	15.27	16.16	16.94	17.60	18.16	18.75
B	3.93	8.99	13.39	16.81	19.50	21.71	23.55	25.01	26.29	27.46	28.44	29.22	29.94	30.57	31.19
CCC/C	28.21	38.67	44.55	48.32	51.13	52.19	53.32	54.15	55.18	55.84	56.47	57.15	57.92	58.54	58.54
Investment grade	0.12	0.33	0.57	0.88	1.19	1.52	1.83	2.13	2.42	2.72	3.00	3.23	3.45	3.66	3.89
Speculative grade	4.13	8.18	11.72	14.58	16.90	18.84	20.47	21.84	23.07	24.17	25.08	25.85	26.54	27.13	27.70
All rated	1.76	3.52	5.07	6.37	7.45	8.39	9.18	9.87	10.50	11.08	11.57	11.98	12.35	12.68	13.01

Ratings of Synthetic Instruments

- For synthetic instruments, such as Collateralized Debt Obligations (CDO), the probability of default is the base for assigning rating
- Typical default definition:

  $$\textit{(cumulative loss)} > \textit{threshold}$$

- There are many model for calculating PD, for example Merton-type models, where company's equity is the threshold

CDO Ratings

Monte-Carlo simulation estimates Probability of Exceedance (POE) of cumulative loss of assets in the CDO, where threshold is the attachment point for a tranche.

Default definition for a CDO tranche

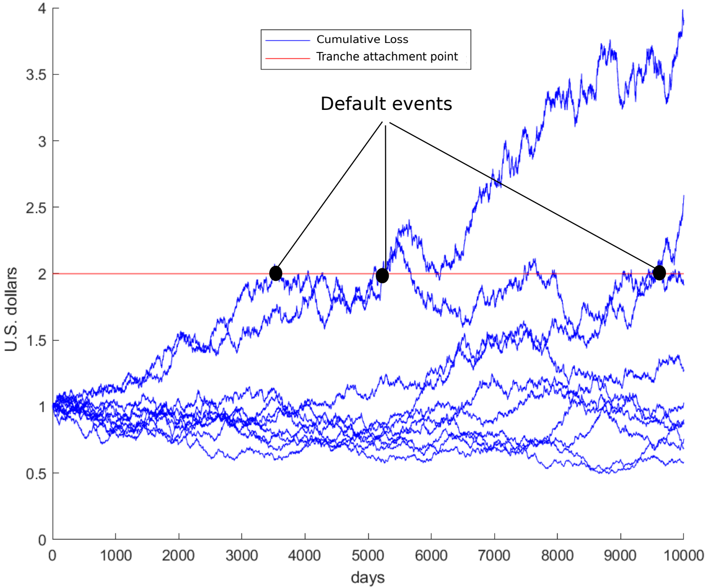
The default of a tranche:

$$L_t > x_m$$

x_m = attachment point of m-th tranche
L_T = cumulative loss at time t

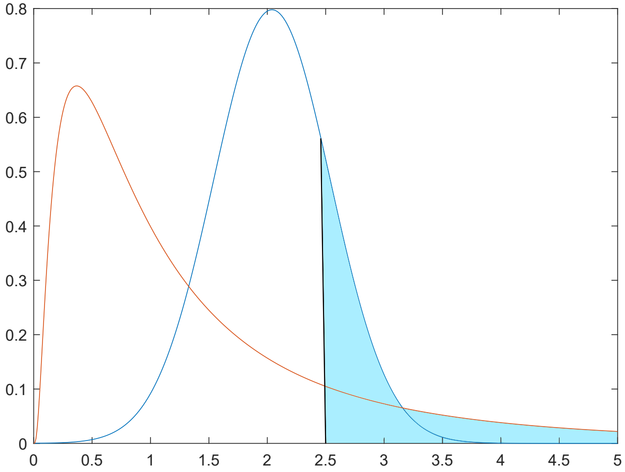
Rating of tranche is assigned based on POE

Monte Carlo Simulation



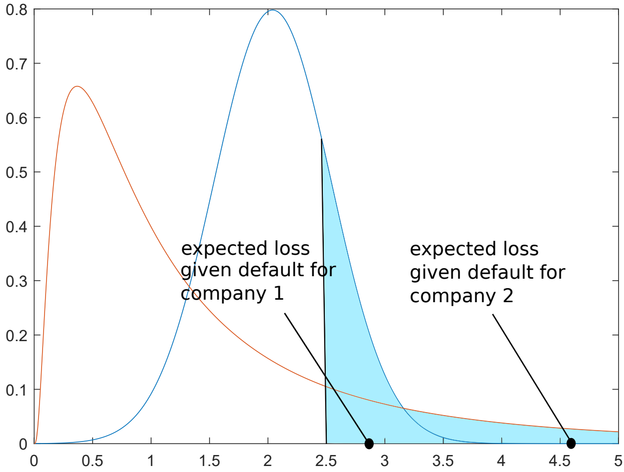
Problems with POE Based Ratings

- Ratings do not account for magnitude of loss in the event of default



Problems with POE Based Ratings (Cont'd)

- Companies with the same default probability might have very distinct loss given default



Solution: bPOE

- For both companies, given threshold of 2.5, we have the same POE = 0.18

- However, bPOE for company 1 with threshold 2.5 is 0.42 and for company 2 is 0.58. This is intuitive because the second company has heavier tail

- bPOE is defined by the average loss in the tail

VaR,POE,CVaR, bPOE

- Constraints on POE and VaR are equivalent
- Constraints on bPOE and CVaR (Expected Shortfall) are equivalent
- VaR based risk methodologies have been supplemented with CVaR based methodologies.
- Similar, POE risk methodologies need to be supplemented with bPOE based methodologies

Definition of bPOE

- POE = 1 - (inverse of VaR)
  bPOE = 1 - (inverse of CVaR)

bPOE formula which can be used as a definition

$$bPOE(z) = \min_{a \geq 0} E(a(X - z) + 1)^{+}$$

Hedge Fund Strategy: Perfect POE Rating

- Sell n_K uncovered european call options, strike $= K$, price $= P(K)$
- $n_K = \frac{1}{P(K)}$ options, income from sale $= \$1$
- Price of underlying $= S_T$ at expiration time $= T$
- Loss at time T, assuming 0 interest rate

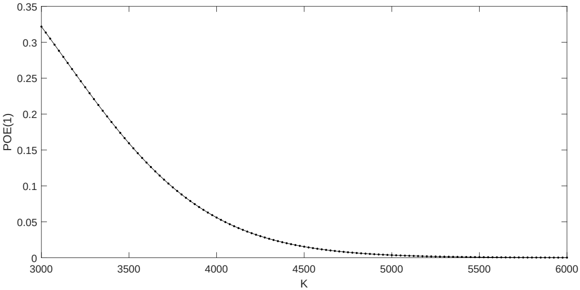
$$L_K = n_K\big(\max\{S_T - K, 0\} - 1\big)$$

  The possible loss in unbounded.

- Default: $\quad L_K > 0$

Hedge Fund Strategy (Cont'd)

- Arbitrary low default probability, high rating for large K
- *Example:* call option, initial value of underlying $= 2700$, annual volatility $= 20\%$, risk free rate $= 5\%$, expiration $= 3$ month. POE depending on prices K:



Solution: bPOE Rating

- Tail of the loss distribution becomes heavier for large n_K
- It can be shown that

$$bPOE_0\left(L_K\right) = 1 \, , \quad \forall \; K$$

while

$$POE_0\left(L_K\right) \to 0 \, , \quad K \to \infty$$

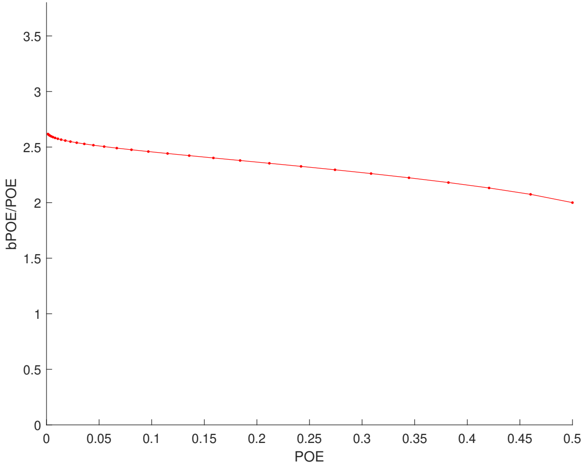
AIG Default

- AIG incurred $25 billion loss due to an extremely large exposure to upper CDO tranches
- U.S. Government had to bailout AIG for $85 Billion. 80% of AIG's equity was bought by the U.S government
- "AIG believed that what it insured would never have to be covered. ... A division of the company, called AIG Financial Products (AIGFP), nearly led to the downfall of a pillar of American capitalism."

  Falling Giant: A Case Study of AIG. Investopedia.com
- **AIG used the Selling Naked Options Hedge Fund Strategy and still kept AAA rating**

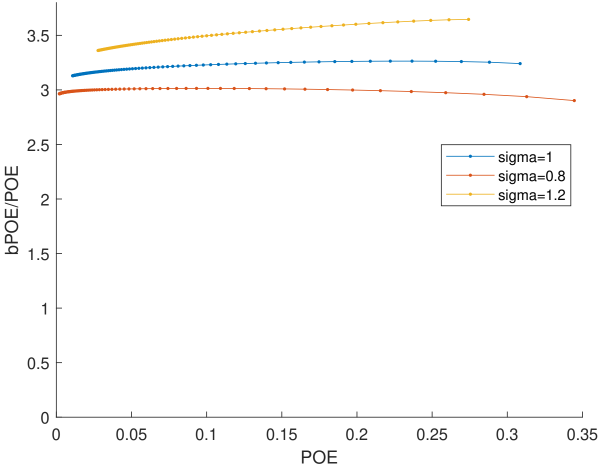
bPOE/POE Ratio for Normal Distribution

- bPOE/POE for Normal distribution as function of POE



bPOE/POE Ratio for Log-normal Distribution

- bPOE/POE for Log-Normal dist. as function of POE



bPOE/POE for Exponential Distribution

- PDF of exponential distribution:

$$p(y) = \lambda e^{-\lambda y}$$

- For exponentially distributed loass L

$$bPOE_x(L)/POE_x(L) = e = 2.718...$$

  for any thresholds x and parameter λ

- Exponential distribution is a "demarcation line" between heavy and light tailed distributions. If a distribution has a heavier tail than exponential distribution with arbitrary parameter λ, then it is called a *heavy tailed distribution*.

bPOE Rating Methodology

- Ratings: POE $=>$ bPOE (with the same threshold)
- POE rating are transformed to bPOE ratings

$$\text{POE} < p_{rating} \quad => \quad \text{bPOE} < e * p_{rating}$$

- bPOE rating is assigned by using scaled POE rating table (probabilities multiplied by e)

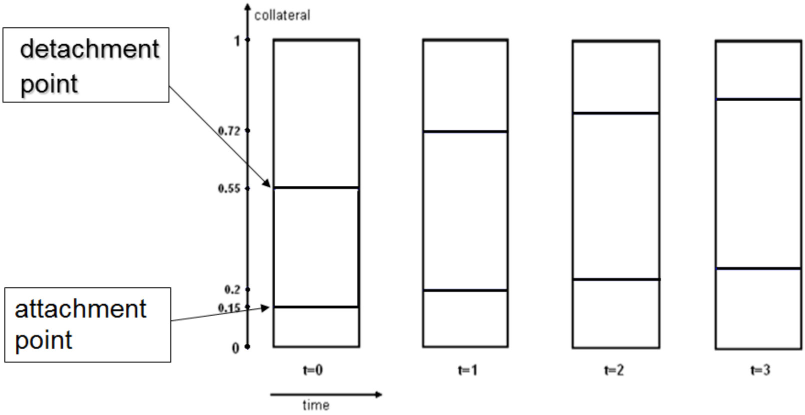
Application of bPOE Ratings: CDO Structuring

- bPOE is a quasy-convex function (level sets are convex)
- Tranche ratings determine the price of a CDO
- CDO structuring: set attachment/detachment points and select the underlying set of assets
- Convex optimization with bPOE constraints, which can be equivalently set as CVaR constraints

Case Study: CDO Structuring

- CDO with 5 tranches
- Time horizon = 5 years
- Interest rate = 7%
- Underlying pool of assets consists of CDS contracts
- CDS pays a fixed spread at the end of each year for 5 years
- The joint loss distribution for CDS pool is generated by the Standard & Poor's CDO Evaluator. 500,000 default scenarios.

CDO attachment/detachment points



CDO Structuring Problem

- Expected payment at time t by CDO to tranche buyers:

$$\sum_{m=1}^{M} s_m \, \mathbb{E}\left[x_{m+1}^t - \max\{x_m^t, L_t\}\right]^+,$$

$$\begin{aligned} M &= \text{number of tranches} \\ s_m &= m\text{-th tranche spread payments} \\ x_m^t &= m\text{-th tranche attachment point at time } t \\ L_t &= \text{total loss of the asset pool at time } t \end{aligned}$$

- Veremyev et al. (2012) reduced to convex function:

$$\sum_{m=1}^{M} s_m \, \mathbb{E}\left[x_{m+1}^t - \max\{x_m^t, L_t\}\right]^+ = \sum_{m=1}^{M} \Delta s_m \, \mathbb{E}\left[x_{m+1}^t - L_t\right]^+,$$

where $\Delta s_m = s_m - s_{m-1}$

CDO Structuring Problem (Cont'd)

- The objective is to minimize the discounted average spread payments made by the CDO issuer

$$\sum_{t=1}^{T} \frac{1}{(1+r)^t} \sum_{m=1}^{M} \Delta s_m \, \mathbb{E}[x_{m+1}^t - L_t]^+$$

CDO Structuring Problem (Cont'd)

- Constraint on total payment obtained from Credit Default Swap (CDS) pool sellers:

$$\sum_{k=1}^{K} c_k y_k \geq \zeta$$

c_k = spread payment of k-th CDS in the pool
y_k = weight of k-th CDS in the pool

- Constraints for tranche bPOE ratings obtained from POE rating with probability p_m

$$bPOE_0(\max(L_t - x_m^1, \ldots, L_t - x_m^T)) \leq e * p_m$$

CDO Structuring Problem (Cont'd)

$$\min_{\boldsymbol{x},\boldsymbol{y}} \sum_{t=1}^{T} \frac{1}{(1+r)^t} \sum_{m=1}^{M} \Delta s_m \, \mathbb{E}\left[x_{m+1}^t - L_t\right]^+$$

$$s.t. \quad bPOE_0\big(\max(L_t - x_m^1, \ldots, L_t - x_m^T)\big) \leq e * p_m$$

$$\sum_{k=1}^{K} c_k y_k \geq \zeta$$

$$L_t = \sum_{k=1}^{K} \theta_k^t y_k$$

$$\sum_{k=1}^{K} y_k = 1$$

$$0 \leq x_1^t \leq \ldots \leq x_m^t \leq 1; \quad y_k \geq 0$$

Case Study: Pareto Frontier

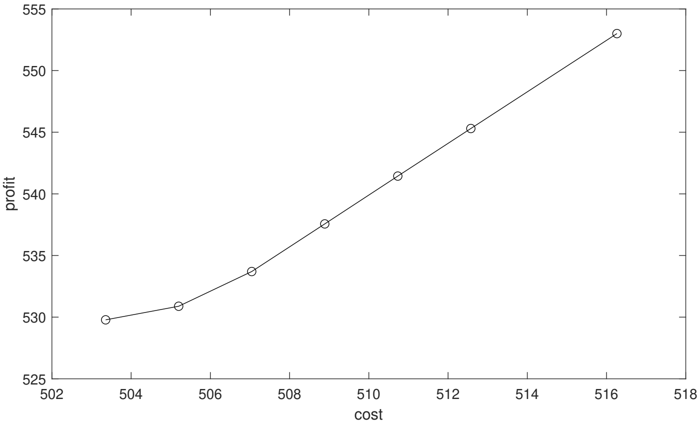


Figure: Pareto frontier of the income and loss for the CDO issuer, horizontal axis = discounted total cost of CDS pool,
vertical axis = discounted total profit of the CDO