Buffered Probability of Exceedance (bPOE) Ratings for Synthetic Instruments

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Abstract. Credit Rating is an important characteristic of company in financial market. Investors determine the appropriate yields (required return) for the assets such as Bonds and CDO tranches, based on credit rating. Current methodology for measuring credit rating for synthetic instruments is based on probability of exceedance concept. The probability of exceedance has several drawbacks as a measure of risk. The most important is that it does not measure the magnitude of loss in the event of default. Therefore, financial instruments with very different exposures in the event of default may have the same rating. This paper illustrates, how the new measure called Buffered Probability of Exceedance (bPOE) can be used to calculate the credit ratings. The bPOE has exceptional qualitative and quantitative characteristics, compared to the probability of exceedance. bPOE is sensitive to the thickness of the tail of the loss distribution. Therefore, the exposure in the event of default impacts the ratings based on bPOE.

Keywords: Buffered Probability of Exceedance, bPOE, Probability of Exceedance, POE, Conditional Value-at-Risk, CVaR, ratings, Collateralized Debt Obligation, CD

1 Introduction

Credit ratings are widely used by investors to assess the credit risk of a security. Currently there are three major credit rating providers (known as "Big Three"): Moody's, Standard and Poor's and Fitch Group. These agencies rate various financial instruments, including so called synthetic instruments. In finance, a synthetic instrument or position, is a way to create the payoff of a financial instrument using other financial instruments. The financial crisis of 2008 showed that credit ratings might not measure the risk appropriately for synthetic instruments such as Collateralized Debt Obligations (CDOs). One such case was the American International Group (AIG), a financial institution that purchased a significant number of CDOs tranches. The U.S. Government had to bailout AIG for $85 billion and bought 80% of its equity. Rating agencies use an approach based on risk measure called Probability of Exceedance (POE). For a random variable X and some threshold x POE is defined as $\mathbb{P}(X > x)$.

This paper suggests a new rating model based on the Buffered Probability of Exceedance (bPOE), that is an improvement compared to the current POE based model. POE based rating do not measure the exposure in the case of default. If one instrument has a heavy-tailed loss distribution, while other one has light-tailed distribution, under POE based rating, they can have the same rating (in some cases, instrument with heavy-tailed loss might even have higher rating). bPOE based rating model will assign lower rating to the instruments with the heavy tails, thus removing incentive to accumulate low default probability but high exposure assets, as it happened with AIG.

2 Current rating models

Rating agencies have been collecting the default statistic of the rated companies for decades. The agencies publish the tables of default probabilities for each rating class over a given time horizon. Table 1 gives the Standard and Poor's default probability table. For example, BBB rating corresponds to a financial instrument with 1 year probability of default (PD) satisfying the inequality 0.08% < PD < 0.23% .

Ratings models for synthetic instruments are quite complicated because of various assumptions and approaches involved in modeling of underlying instruments. However, the approach for issuing the rating, when simulation model is built, is quite simple. We will explain this approach with an example based on the Merton model. Suppose that a firm finances its operation by issuing a single zero-coupon bond with face value B_T payable at time moment T. It is assumed that at every time moment $t \in [0, T]$, the company has total assets A_t, following Geometric Browning motion dynamics.

Merton model assumes that the default of the company occurs when the firm has no capital (equity) to pay back the debt holders. Because the only payment the zero-coupon bond makes is at time T, that is the only moment when the default can occur. It is straightforward to calculate the probability of default (PD) for a given firm. The probability of default at time T is

$$\mathbb{P}(default) = \mathbb{P}(A_T < B_T) \quad (1)$$

Formula (1) can be rewritten in terms of POE by changing the sign of assets and liabilities,

$$\mathbb{P}(default) = \mathbb{P}(A_T < B_T) = \mathbb{P}(-A_T > -B_T)$$

Thus, PD is a POE of random variable $-A_T$ with threshold $-B_T$. Table 1 can be used to convert the PD calculated using the Merton model into a rating (e.g., if 1 year PD satisfies inequality 0.08% < PD < 0.23%, then the company has BBB rating). Despite unrealistic assumptions, the Merton model provides the base for more complex models which are widely used in the industry.

Average Cumulative Default Rates For Corporates By Region (1981-2015) (%)

	--Time horizon (years)--														
Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
U.S.															
AAA	0.00	0.04	0.17	0.29	0.42	0.54	0.59	0.67	0.76	0.86	0.90	0.95	1.00	1.10	1.21
AA	0.04	0.08	0.18	0.32	0.46	0.61	0.76	0.88	0.98	1.09	1.19	1.28	1.37	1.45	1.55
A	0.08	0.21	0.37	0.56	0.75	0.97	1.22	1.45	1.70	1.95	2.18	2.38	2.58	2.75	2.95
BBB	0.23	0.61	1.02	1.54	2.10	2.65	3.15	3.65	4.15	4.64	5.12	5.50	5.86	6.23	6.60
BB	0.81	2.51	4.58	6.60	8.38	10.14	11.61	12.96	14.17	15.27	16.16	16.94	17.60	18.16	18.75
B	3.93	8.99	13.39	16.81	19.50	21.71	23.55	25.01	26.29	27.46	28.44	29.22	29.94	30.57	31.19
CCC/C	28.21	38.67	44.55	48.32	51.13	52.19	53.32	54.15	55.18	55.84	56.47	57.15	57.92	58.54	58.54
Investment grade	0.12	0.33	0.57	0.88	1.19	1.52	1.83	2.13	2.42	2.72	3.00	3.23	3.45	3.66	3.89
Speculative grade	4.13	8.18	11.72	14.58	16.90	18.84	20.47	21.84	23.07	24.17	25.08	25.85	26.54	27.13	27.70
All rated	1.76	3.52	5.07	6.37	7.45	8.39	9.18	9.87	10.50	11.08	11.57	11.98	12.35	12.68	13.01

Table 1. PD as a function of rating (published by Standard and Poor's). The PD is measured over a given time horizon.

3 bPOE Ratings

We propose a new methodology for assigning ratings to synthetic instruments, based on the bPOE concept. bPOE with threshold v for a random variable X is equal to the probability of the right tail of the distribution of X such that the average of this tail is equal to v. Formally, bPOE can be defined as follows (see, [1]),

$$bPOE(v) \;=\; \min_{a\geq\,0} \mathbb{E}[a(X-v)+1]^{+} \tag{2}$$

where $[x]^{+} \;=\; max\{x,0\}$. bPOE is equal to one minus inverse of Conditional Value-at-Risk (CVaR), where CVaR in the average of the tail having the probability $1-\alpha$. Formally, CVaR is defined as follows (see, [5]),

$$CVaR(\alpha) = \min_{C} \left(C + \frac{1}{1-\alpha} \mathbb{E}[X-C]^{+} \right)$$

By definition bPOE equals POE = $1-\alpha$ of the right tail with $CVaR(\alpha)=v$, see Figure 1.

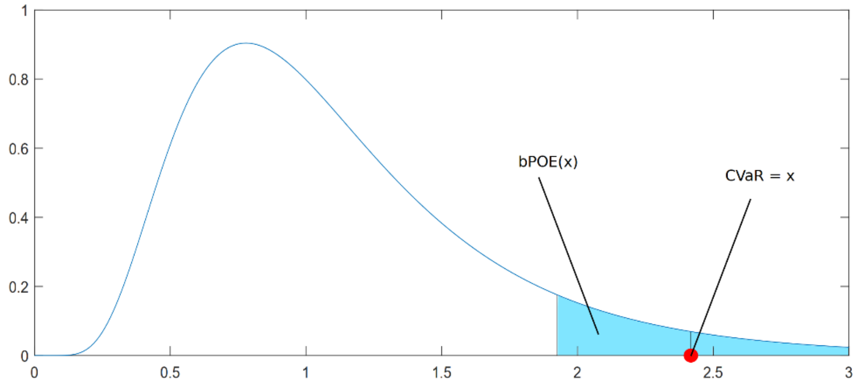

Fig. 1. Relationship of bPOE and CVaR (the shaded region area is equal to bPOE)

For more information about properties of bPOE see [1]. Note, the formula (2) is considered a property of bPOE in paper [1], however, it is convenient to use it as a definition.

For evaluation of ratings, we suggest to replace POE with bPOE, calculated for the same threshold. bPOE, by construction, is always greater than the POE with the same threshold. For example, for the standard normal distribution, bPOE is roughly 2.4 times higher than the POE with the same threshold. For the log-normal distribution with parameters $\mu = 0$ and $\sigma = 1$, bPOE is roughly 3.2 time higher than POE. We propose to rescale the probabilities in the rating tables, by bPOE/POE ratio calculated for the exponential distribution. bPOE ratings will be calculated using the intervals from the new table. There are two reasons why exponential distribution is a good candidate for rescaling:

1. Exponential distribution is the "demarcation line" between heavy-tailed and light-tailed distributions. The distribution is called heavy-tailed if

$$\lim_{v \to \infty} e^{\lambda v} \, \mathbb{P}(X \geq v) = \infty \ , \quad \forall \lambda > 0.$$

   I.e., heavy-tailed distribution has heavier tails than the exponential distribution with arbitrary parameter λ.

2. The bPOE(v)/POE(v) ratio for the exponential distribution with arbitrary parameter $\lambda > 0$ and arbitrary threshold value $v > \mathbb{E}\, X$, is constant and equal to e = 2.718.... . bPOE for the exponential distribution (see [2]) equals $e^{1-\lambda v}$. The POE for the exponential distribution equals $e^{-\lambda v}$, thus, the ratio of bPOE to POE equals e.

4 Case Study

This section illustrates bPOE ratings for a CDO's loss distribution. The data comes from paper [3]; the dataset can be downloaded from [4]. This data represents a loss distribution of the underlying assets of the CDO over the period of one year, generated by Standard & Poor's CDO Evaluator. Figure 2 shows the histogram of the loss distribution.

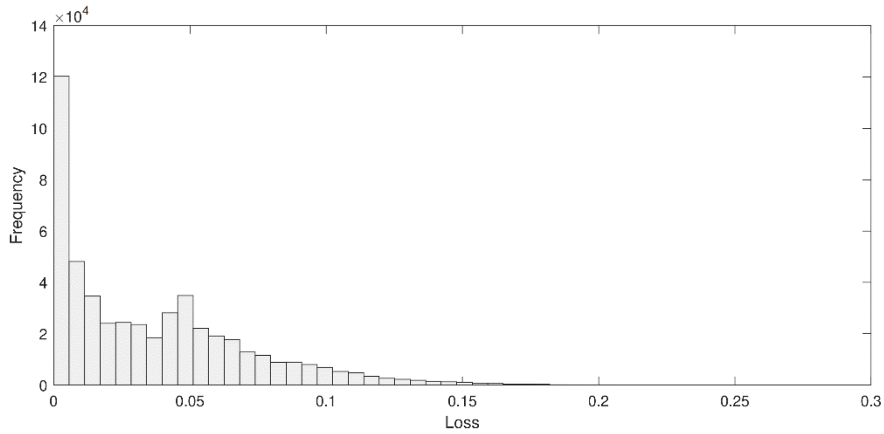

Fig. 2. The histogram of the loss distribution

We take threshold values of 0.12 and 0.15 to illustrate bPOE ratings. The POEs for the threshold values 0.12 and 0.15 are 2.73% and 0.8% respectively. Based on the Standard and Poor's ratings, 2.73% falls into the B rating bracket, because it is within the following interval 0.81% < 2.73% < 3.93% and 0.8% fall into the BB rating bracket, because it is within the following interval 0.23% < 0.8% < 0.81%.

The bPOEs for 0.12 and 0.15 threshold values are 7.24% and 2.06%. By scaling the probabilities in the Table 1, using the exponential distribution coefficient e, we get that the ratings are not changed for this case. The bPOE corresponding to the 0.12 threshold value falls in the interval 2.718*0.81*% < 7.24% < 2.718*3.93% and bPOE for 0.15 falls in the interval 2.718 *0.23%< 2.06% <2.718*0.81%, corresponding to B and BB ratings respectively. The fact that ratings are unchanged means that the loss distributions tail is similar to the tail of the exponential distribution. Therefore, the bPOE ratings are close to the POE ratings for this dataset.

5 Summary

The paper presented the application of bPOE for defining the credit ratings. bPOE accounts for information about magnitude of the losses in the tail of the distribution. The paper proposed coefficient equal to e for converting existing rating probability bounds into the bPOE rating probability bounds. The conversion factor is based on the assumption that losses are distributed exponentially. Any loss distribution that has heavier tails than the exponential distribution (and thus is a heavy tailed distribution), will have lower bPOE rating than the POE based rating.

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