Introduction

Intermediation Trends

- Large investors increasingly prevalent in securities markets

Equity investments in the U.S. Source: The Economist.
Vayanos’ View

“If most large trades were motivated by information, large traders would significantly outperform the market. However, many empirical studies show that large traders do not significantly outperform, and may even underperform, the market. [...] Therefore, allocation motives must be important.”

Source: Vayanos (2001, JF, p. 132)
Large Orders Impact Price

Sell orders due to transition from legacy portfolio

Source: Obizhaeva (2009, Fig. 1).
Large Orders Impact Price

Price reverts after execution ends

Price Impact During Execution

Price sometime reverts before execution ends

Source: Zarinelli, Treccani, Farmer, and Lillo (2015), Figure 8.
Participation Rate $\pi$ and Duration $D$ are negatively correlated.

Source: Zarinelli, Treccani, Farmer, and Lillo (2015), Figure 2.
Regulatory Pressure

- Large broker-dealers might be forced to sell quickly to meet minimum liquidity ratio (Basel III)
- Cover-2 capital requirement for CCPs forces them to assess liquidity premium paid when positions of a failed account need to be sold in “close-out period.”
- SEC (2016) demands open-end funds to report their liquidity risk in terms of “days-to-cash”.
- Massive liquidations during the current COVID 19 pandemic: money market funds had to liquidate large parts of their portfolios to meet liquidity requirements (see BIS (2020))
Regulatory Pressure

**Figure:** MSCI Liquidity Surface
Papers on optimal execution with exogenous liquidity supply (e.g., Almgren and Chriss (2001)).

Papers on optimal liquidity supply with exogenous demand (e.g., Amihud and Mendelson (1980), Hendershott and Menkveld (2014)).

Very few papers on both (e.g., Pritsker (2009), Rostek and Weretka (2015))
Research Questions

- How liquid is the market for a large seller who is (only) time constrained?
- Should he reveal this constraint?
- Do market makers benefit from the large-seller’s presence? And, end-user investors?
- Calibrate the model to assess economic size of these effects
Model Setting

- Strategic trading by large seller who needs to trade a large position in finite time.
- Strategic trading by (Cournot) competitive market makers in response to large seller (Stackelberg)
- Information symmetry on fundamentals
- Time is continuous and runs forever
Model Visualization

- Trades
  - buy end-investor
  - sell end-investor

- Position
  - Position market makers

- Price
  - Ask price
  - Bid price

- Time
  - Sell rate large seller

The Model

Agostino Capponi, Albert Menkveld, Hongzhong Zhang
Large Orders in Small Markets: On Optimal Execution
May 15, 2020
The security’s fundamental value $m$ is common knowledge:

$$dm_t = \sigma dB_t,$$

where $B_t$ is a standard BM.
End-User Investors

- Buy and sell end-user investors arrive according to a Poisson process $N^a_t$ and $N^b_t$ with the same intensity $\lambda > 0$.
- Upon arrival, the traded amount depends on ask and bid prices.
  - For buyers, it is $q^a_t = \delta (\omega - p^a_t)$, where
    - $\omega$ is the end users’ reservation value
    - $p^a_t$ is the ask price pressure, defined as the ask quote minus $m_t$
  - For sellers, it is $q^b_t = \delta (p^b_t + \omega)$
Large Seller

- Seller arrives at time $t = 0$
- He privately observes his duration $D \sim \text{Exp}(\nu)$
- Let $p^b_t$ be the bid price pressure offered by market makers at time $t$. The large seller picks the trade intensity $f$:

$$\sup_{f \geq 0} \mathbb{E} \left[ \int_0^D e^{-\beta t} f \times (\omega + p^b_t) dt \right]$$

- In the process, we analyze two situations for the large seller
  - **Stealth trading**: keeps $D$ hidden
  - **Sunshine trading**: implicitly reveals $D$
Market Makers

- $N$ market makers share the liquidation stream from the large seller.
- Market maker $n$ chooses how much to buy at the bid ($q_{nt}^b$), and how much to sell at ask ($q_{nt}^a$), to maximize revenues.
- The aggregated strategies of the $N$ market makers collectively determine the ask and bid prices:

$$\left\{ \begin{array}{l}
N\sum_{n=1}^{N} q_{nt}^a = \delta (\omega - p_t^a) \\
N\sum_{n=1}^{N} q_{nt}^b = \delta (p_t^b + \omega)
\end{array} \right.$$ 

$$\Rightarrow p_t^a = \omega - \frac{1}{\delta} \sum_{n=1}^{N} q_{nt}^a \quad p_t^b = \omega + \frac{1}{\delta} \sum_{n=1}^{N} q_{nt}^b$$
The Objective of Market Makers

- Market makers compete a-la-Cournot. Market maker $n$ picks trading quantities:

$$
\max_{(q^a_n, q^b_n | q^-_n, q^b^-_n)} \mathbb{E} \left[ \int_0^\infty e^{-\beta t} (dW_{nt} - \eta (i_{nt})^2 \, dt) \right],
$$

where

$$
dW_{nt} = -p^b_t \left( \frac{f}{N} 1_{t \leq D} + q^b_{nt} dN^S_t \right) + p^a_t q^a_{nt} dN^B_t + m_t d\hat{i}_{nt},
$$

$$
d\hat{i}_{nt} = \frac{f}{N} 1_{t \leq D} dt + q^b_{nt} dN^S_t - q^a_{nt} dN^B_t
$$

- Shares liquidated by large investor
- Shares bought from sell investors
- Shares sold to buy investors

- Focus on symmetric equilibria
Finding equilibria

- We solve the Stackelberg game backwards. Pick duration $D$
  1. Fix $f$ and recover the equilibrium trading quantities $q^a_n, q^b_n$, $n = 1, \ldots, N$, by solving the market maker optimization problem
  2. Solve for optimal $f$, accounting form market makers’ responses.
- Baseline case of no seller is a by-product (only second stage game)
- Stealth and sunshine differ by revealing or not $D$ to market makers
Baseline (no large seller)

**Proposition 1:**

- There is a unique equilibrium
- Ask and bid prices are

\[
p^a(i) = A_\theta - B_\theta Ni
\]
\[
p^b(i) = -A_\theta - B_\theta Ni,
\]

where \( \theta = (\lambda, \eta, \beta, \omega, \delta, N) \).

- The associated depths are

\[
q^a(i) = C_\theta + D_\theta i
\]
\[
q^b(i) = C_\theta - D_\theta i
\]
Sunshine Equilibrium

Proposition 2:

- If there is a large seller who engages in sunshine trading, then

  - There is a unique equilibrium
  - Bid and ask prices are

\[
\begin{align*}
p^{a,S}(i, f, t) &= p^a(i) - 1_{t \leq D} f(G^S_\theta(D - t)) \\
p^{b,S}(i, f, t) &= p^b(i) - 1_{t \leq D} f(G^S_\theta(D - t) + H^S_\theta)
\end{align*}
\]

The associated depths are

\[
\begin{align*}
q^{aS}(i, f, t) &= q^a(i) + 1_{t \leq D} f \left( \frac{\delta}{N} (G^S_\theta(d - t)) \right), \\
q^{bS}(i, f, t) &= q^b(i) - 1_{t \leq D} f \left( \frac{\delta}{N} (G^S_\theta(d - t) + H^S_\theta) \right)
\end{align*}
\]
Expected Price Paths Sunshine Trading

- Price pressure: the deviation from fundamental
Sunshine vs Stealth

Proposition 4:

- The large seller prefers, on average, sunshine over stealth.
- Even if he is constrained to trade at a fixed intensity.
- The benefits under sunshine are even higher, because the large seller can make the liquidation rate conditional on duration \( D \).
- The large seller is better off even in the case of a monopolistic market maker.
Lemma 5: Let $f^{sun}$ and $f^{sth}$ denote the optimal liquidation intensity of the large seller under sunshine, respectively, stealth trading. Then

- $f^{sun}(D)$ is continuous and strictly decreasing in $D$ and
- the stealth intensity is wedged between the lowest and highest sunshine intensity:

\[ 0 < \lim_{D \to \infty} f^{sun}(D) < f^{sth} < f^{sun}(0) = \frac{N\delta\lambda \omega}{2} < \infty. \]
**Lemma 1**: Under sunshine trading, market makers benefit from the presence of a large seller for any liquidation intensity $f$ if

- Either the risk-bearing capacity of the market is sufficiently high (i.e., low inventory costs, high arrival rate of end investors, and enough market makers), or
- duration of the large seller’s trade is short.
Impact on End Users

- Economic surplus of end users:

\[
U(t, i; \bar{f}) = \mathbb{E}_{t=i} \left[ \int_t^\infty e^{-\beta(s-t)} \left( \frac{1}{2} \delta (\omega - \tilde{a}_s)^2 dN_s^B + \frac{1}{2} \delta (\omega + \tilde{b}_s)^2 dN_s^S \right) \right]
\]

**Proposition 5:** Liquidation benefits end users if \( \bar{f} \) is “high enough”, i.e.,

- \( U(0, 0, \bar{f}) - U(0, 0, 0) < 0 \) if \( \bar{f} \) is below a threshold
- \( U(0, 0, \bar{f}) - U(0, 0, 0) > 0 \) if \( \bar{f} \) is above a threshold.
Welfare Impact of Liquidation

Impact on End Users: Intuition

- **High liquidation rate:**
  - The additional price pressure benefits end users (Hendershott and Menkveld (2014))
  - But, the model predicts that liquidation may widen the bid-ask spread, which harms end users investors' surplus

- **Low liquidation rate:**
  - Execution costs due the widened bid-ask spread dominate the positive effects due to intensified price pressure.
Market makers are HFTs.

Matches HFT trading in Euronext stocks (Menkveld (2013))

\[
\begin{align*}
N &= 10, \\
\eta &= 0.140802 \text{ (bps/€1000)}, \\
\delta &= 11.2885 \text{ (€1000/bps)}, \\
\omega &= 13.61 \text{ (bps)}, \\
\lambda &= 791 \text{ (/day)}, \\
\nu &= 5 \text{ (/day)}. 
\end{align*}
\]
Calibration Results

Large seller's sell intensity (€10^6/day)

- Sunshine equilibrium
- Stealth equilibrium

Sunshine minus stealth liquidation proceeds (€/day)

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Figure: Calibrated model visualization.

This figure illustrates the model by plotting the mean of various model variables based on the calibration. The top graph plots the (continuous) net flow of the seller as well as the mean arrival rate of buyers and sellers. The middle graph plots market makers’ inventory which result from the net flows in the top graph (assuming they start off at zero inventory). The bottom graph plots the bid and ask prices relative to fundamental value.
Sunshine vs Stealth

- 4.188% improvement when large seller switches from stealth to sunshine, but maintains the same liquidation rate
- An additional 1.266% improvement is obtained if the seller picks the liquidation rate that is optimal for each $D$
Social Efficiency

Proposition: If instead of the market makers, a social planner sets prices then the Stackelberg equilibrium changes in the following ways, compared to sunshine and to stealth:

- The bid-ask spread is strictly lower.
- The spread is no longer elevated in the liquidation period. It becomes a liquidation-invariant constant.
- In the sunshine case, the large-seller trades at higher intensity for short durations. In fact, rather than being finite the limit intensity is infinite when the seller’s duration $D$ is taken to zero:

$$\lim_{D \downarrow 0} f^{\text{Planner}}(D) = \infty.$$
Social Efficiency

Large seller's sell rate ($10^6$/day)

- Sunshine equilibrium
- Constrained first-best

Duration (days)
A direct comparison between the constrained first-best and the sunshine equilibrium yields: ‘

- Seller sells, on average, at almost 4 times higher intensity
- The bid-ask spread drops from 4 to 0.03 basis points
- Yet, welfare is only 1.4% higher (vs. 11.1% for duopoly and 24.4% for monopoly)
**Figure:** MSCI liquidity surface
CMZ Liquidity Surface
### Conclusions

- Liquidation reinforces price pressure and widens bid-ask spread
- Sunshine dominates stealth, on average across durations
- Large seller’s presence does not unambiguously benefit all others
- Model generates empirical patterns: most strikingly, price reversal prior to the end of large seller’s liquidation


Extensions

- Enlarge strategy space and allow for predation under sunshine trading
- What if the liquidation rate is allowed to depend on end-user arrivals?
<table>
<thead>
<tr>
<th></th>
<th>Menkveld (2013)</th>
<th>Our model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cond. price pressure (bps/€1000)</td>
<td>-0.026</td>
<td></td>
</tr>
<tr>
<td>Std. inventory (€1000)</td>
<td>80.3</td>
<td></td>
</tr>
<tr>
<td>Half bid-ask spread (bps)</td>
<td>1.6</td>
<td>$\tilde{\rho} \sqrt{\frac{2cA}{2A(N+1)}}$</td>
</tr>
<tr>
<td>Mean average No. arrivals (/day)</td>
<td>1582</td>
<td>$\frac{2NA}{N+1+2cA}$</td>
</tr>
</tbody>
</table>

$$\tilde{\rho}$$
Participation Rate vs Duration

Rate and Duration are negatively correlated.

Source: Zarinelli, Treccani, Farmer, and Lillo (2015), Figure 2.
Regulatory Pressure

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Participation Rate

- Participation rate is the liquidated volume over the total volume until $D$:

$$R(D) = \frac{D \cdot f^*(D)}{\mathbb{E}[\text{total volume}]}$$

- We show that $1/R(D)$ is strictly increasing in $D$:
- Thus, **the participation rate strictly decreases with** $D$. 
Impact on Market Makers

- Suppose market makers start with zero inventory. Then, the value of one market maker is $C(0, \bar{f})$ under stealth and $\tilde{C}(0, \bar{f})$ under sunshine.

- Suppose the following liquidity condition holds:

$$N^2 + cA(2N - 1) - \frac{1}{2}(1 + cA)(N + 2cA)^2 \left(1 - \frac{\beta^2}{\delta^2}\right) \geq 0,$$

- The above condition holds in a liquid market: low inventory costs $\Theta$ or high arrival rate $\lambda > 0$.

- Liquidation **always benefits** the market maker, i.e., $C(0, \bar{f}) > C(0, 0)$ and $\tilde{C}(0, \bar{f}) > \tilde{C}(0, 0)$
Impact on End Users

- Economic surplus of end users:

\[
U(t, i; \bar{f}) = \mathbb{E}_{t=i} \left[ \int_t^\infty e^{-\beta(s-t)} \left( \frac{1}{2} c(\tilde{p} - \tilde{a}_s)^2 dN^B_s + \frac{1}{2} c(\tilde{p} + \tilde{b}_s)^2 dN^S_s \right) \right]
\]

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- **Low liquidation rate:**
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Expected Proceeds of Large Seller

- Under sunshine trading, the large seller’s expected proceeds, given $D$, are

$$
\mathbb{E}^{N^S, N^B} \left[ \int_0^D e^{-\beta t} \bar{f} b_t dt \right],
$$

- Under stealth trading, the expected proceeds are

$$
\mathbb{E}^{D, N^S, N^B} \left[ \int_0^D e^{-\beta t} \bar{f} b_t dt \right].
$$
If \( I_0^{(x^n, n)} = 0 \), the expected inventory at \( t \leq D \) is given by

\[
g(t) \equiv \mathbb{E}[I_t^{(x^n, n)}] = \frac{\bar{f}}{N} \frac{N + 2cA}{N + 1 + 2cA} \left( \frac{\beta}{\delta} \frac{1 - e^{-Mt}}{M} + \frac{\delta - \beta}{\delta} \frac{e^{\delta t} - e^{-Mt}}{M + \delta} e^{-\delta S} \right),
\]

where \( \delta = \Theta/A \). For \( t > D \), \( g(t) = g(D)e^{-M(t-D)} \).

Recall that the expected ask and bid prices are

\[
\begin{align*}
\mathbb{E}[a_t(i, \bar{f})] &= S_0 + \frac{p(1 + 2cA) - 2NAG(t) + NB(t, \bar{f})}{N + 1 + 2cA} \\
\mathbb{E}[b_t(i, \bar{f})] &= S_0 + \frac{-p(1 + 2cA) - 2NAG(t) + NB(t, \bar{f}) - \frac{\bar{f}}{c\lambda} 1_{t \leq D}}{N + 1 + 2cA}
\end{align*}
\]
Simulated Price Pressures

(a) Simulated Price Pressures

(b) Inventory
(c) Mid-quote pressure when $D = 0.2$

(d) Mid-quote pressure when $D = 0.05$
Impact of Liquidation on Others

Box-plots Inventories

(e) Inventory when $D = 0.2$

(f) Inventory when $D = 0.05$