Case Study: Style Classification with multiple Co-Conditional Drawdown-at-Risk Regression

1 Background

This case study applies CVaR regression to the drawdown-based style classification of a mutual fund. The procedure regresses fund cumulative drawdowns by four indices' cumulative drawdowns as explanatory variables. The estimated coefficients represent the fund's style with respect to each of the indices. This problem is inspired by Bassett and Chen (2001) where they conducted style analysis of quantiles of the return distribution. Their approach is based on the quantile regression approach suggested by Koenker and Bassett (1978). In our case study we perform a CVaR regressesion of the drawdowns of the Fidelity Magellan Fund on drawdowns of the Russell Value Index (RUJ), RUSSELL 1000 VALUE INDEX (RLV), Russell 2000 Growth Index (RUO) and Russell 1000 Growth Index (RLG). This multiple regression problem (Problem 1) is developed studied in Ding and Uryasev (202), where the approach is called multiple Co-Conditional Drawdown-at-Risk and is also applied to systemic risk contribution measurement tasks[1]. Here, we want to calculate coefficients for the explanatory variables of the tail of the distribution of residuals (these coefficients may differ from the regression coefficients for the mean and the median of the distribution). The confidence level in our regression problems are 0.9 (tail of large drawdowns) and 0.0 (average drawdowns).

The mCoCDaR regression coefficients is estimated with the linear regression minimizing the CVaR (Superquantile) error, as defined by Rockafellar et al (2014), of drawdowns instead of returns. This error is called CVaR2_err in PSG. Problem 1 minimizes CVaR2_err function for two different risk levels, 0.9 (tail average of large drawdowns) and 0.0 (average drawdowns). Problem 2 and 3 are adopted from previous case studies which used quantile regression[2] and CVaR regression[3] on the returns to perform fund classification for the same task.

2 Statistical Problems

2.1 Drawdown Definition

Suppose $X_1, \ldots, X_T$ are the rates of return of a risky instrument coming from a distribution of return random variable X. Let ξ_t be the cumulative rate of return of the instrument for time t, which can be either uncompounded and defined by $\xi_t = \sum_{k=1}^{t} X_k$. The drawdown of the instrument at time t is defined by

$$Y_t = \max_{1 \leq k \leq t} \xi_k - \xi_t, \qquad t = 1, \ldots, T \tag{1}$$

2.2 Estimation of CoCDaR

The Conditional Drawdown-at-Risk (CDaR) for return random variable X is equivalent to the Conditional Value-at-Risk (CVaR) of its drawdown random variable Y:

$$CVaR_\alpha(Y) = CDaR_\alpha(X)\ .$$

[1]http://uryasev.ams.stonybrook.edu/index.php/research/testproblems/financial_engineering/case-study-cocdar-approach-systemic-risk-contribution-measurement

[2]http://uryasev.ams.stonybrook.edu/index.php/research/testproblems/financial_engineering/style-classification-with-quantile-regression/, problem 1

[3]http://uryasev.ams.stonybrook.edu/index.php/research/testproblems/financial_engineering/on-implementation-of-cvar-regression/, problem 1, alpha=0.9

Where CVaR of random variable X, see Rockafellar and Uryasev (2002), can be calculated as:

$$CVaR_\alpha(X) = \min_C \{C + \frac{1}{1-\alpha} E[(X-C)^+]\} ,$$

and $A^+ = \max\{0, A\}$.

Let D_{fund} define the drawdown random variable of a fund and let the drawdown variables of factor indices $i = 1, \ldots, I$ be denoted as D_i. Given a sample path of data $\{D_{fund,t}, D_{1,t}, \ldots, D_{I,t}\}_{1 \le t \le T}$,, consider the following regression which is also used by Adrian and Brunnermeier (2008) and Huang and Uryasev (2017):

$$D_{fund,t} = \beta_0 + \beta_1 D_{1,t} + ... + \beta_I D_{I,t} ,$$

We define the residual random variable as:

$$L = D_{fund} - (\beta_0 + \beta_1 D_i + ... + \beta_I D_{I,t}) .$$

The estimate of the α-CVaR of D_{fund} can be obtained by minimizing the CVaR (superquantile) error function from Rockafellar et al (2014):

$$\mathcal{E}_\alpha^{CVaR}(L) = \frac{1}{1-\alpha} \int_0^1 CVaR_\gamma^+(L)\, d\gamma - E[L] . \tag{2}$$

It has been proved by Golodnikov et al (2019) that minimization of error (2) for CVaR regression can be reduced to the minimization of the Rockafellar error (convex and liner programming formulations are in Appendix A, Golodnikov et al (2019)). The Rockafellar error belongs to the Mixed Quantile Quadrangle, as defined by Rockafellar and Uryasev (2013). For given confidence levels $\alpha_k \in (0,1)$ and wights $\lambda_k > 0$, $k = 1, \ldots, K$ such that $\sum_{k=1}^{K} \lambda_k$=1, the Rockafellar error equals:

$$\mathcal{E}^{ROC}(L) = \min_{C_1,\ldots,C_K} \left\{ \sum_{k=1}^{K} \lambda_k \mathcal{E}_{\alpha_k}^{KB}(L - C_k) \mid \sum_{k=1}^{K} \lambda_k C_k = 0 \right\} , \tag{3}$$

where the rescaled Koenker-Bassett (KB) error equals:

$$\mathcal{E}_\alpha^{KB}(L) = E\left[\frac{\alpha}{1-\alpha} L^+ + (-L)^+\right] . \tag{4}$$

Denote by $\hat{\beta}_0^\alpha, \hat{\beta}_1^\alpha, ..., \hat{\beta}_I^\alpha$ the regression coefficients obtained by minimizing Rockafellar error (3). The Conditional Drawdown-at-Risk of the fund can be estimated as:

$$CDaR_{t,\alpha}^{fund} = \hat{\beta}_0^\alpha + \hat{\beta}_1^\alpha D_{i,t} + ... + \hat{\beta}_I^\alpha D_{I,t} .$$

The estimated coefficients represent the fund's style with respect to each of the indices, and in terms of its drawdown behaviors at different risk levels. We compare the resulting coefficient estimates with those obtained using quantile regression and CVaR regression on returns of the fund and indices.

3 Optimization Problems

3.1 Optimization Problem 1

Minimizing CVaR (Superquantile) error of drawdowns:

$$\hat{\beta}^\alpha = argminCVaR2\_err(L)$$

3.2 Optimization Problem 2

Minimizing Koenker and Basset error of returns:

$$\hat{\beta}^\alpha = argminKB\_err(L)$$

3.3 Optimization Problem 3

Minimizing CVaR (Superquantile) error of returns:

$$\hat{\beta}^{\alpha} = argminCVaR2\_err(L)$$

4 References

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