Drawdown Beta and Portfolio Optimization

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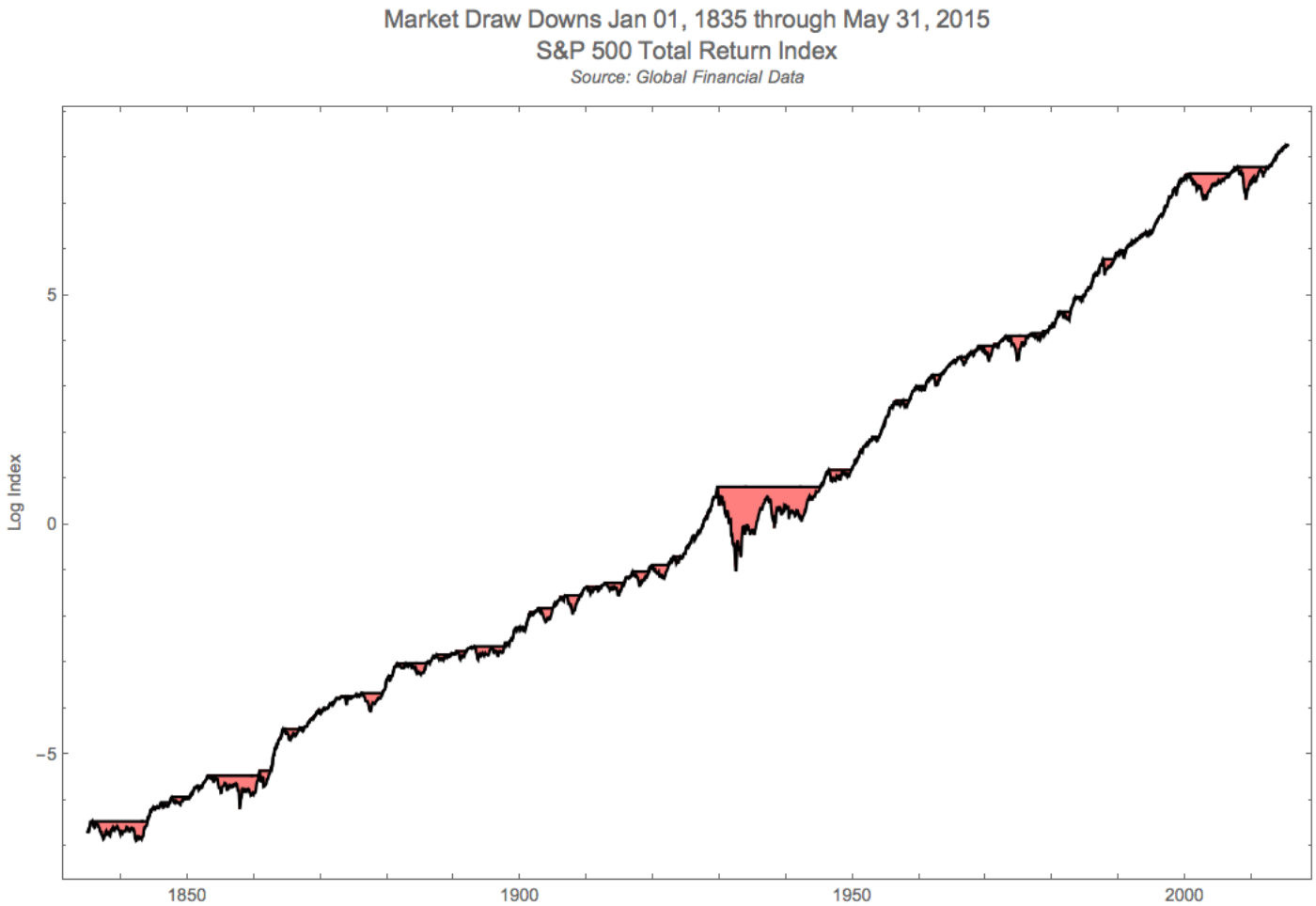
Relevant Links

- Stan Uryasev website at Stony Brook University:
  http://uryasev.ams.stonybrook.edu
- Publications:
  1. Ding, R. and S. Uryasev. Drawdown Beta and Portfolio Optimization (2021):
     http://uryasev.ams.stonybrook.edu/wp-content/uploads/2021/10/Drawdown_Portfolio_Optimization_Problems_and_Drawdown_Betas.pdf
  2. Zabarankin M., Pavlikov K. and S. Uryasev. Capital Asset Pricing Model (CAPM) with Drawdown Measure (2014):
     http://uryasev.ams.stonybrook.edu/wp-content/uploads/2014/01/CDaR-CAPM-published.pdf
- Drawdown Beta website:
  http://qfdb.ams.stonybrook.edu/index_SP_10.html
- CDaR portfolio optimization case study:
  http://uryasev.ams.stonybrook.edu/index.php/research/testproblems/financial_engineering/portfolio-optimization-with-drawdown-constraints-on-a-single-path/

Drawdown: Static-Dynamic Risk Measure

- Drawdown measures current portfolio value compared to the previous pick value
- Compared to other popular risk measures, such as variance and Value-at-Risk, it shows possible losses over several consecutive periods
- Drawdown is a so called static-dynamic risk measures: 1) it is dynamic because many time periods are considered; 2) It is static in the sense that decisions are not made on every step of the dynamic process
- Popular in active portfolio management

SP500 Drawdowns: 1835-2015



Standard Beta

- Standard Beta = normalized correlation of returns of an instrument and the market
- Beta is considered in the framework of Capital Asset Pricing Theory
- Beta equations can be interpreted as a necessary condition of extremum in Markowitz mean-variance portfolio optimiation problem
- Instrument with negative beta is a statistical hedge (protection) which supposed to generate positive returns when market goes down
- However, correlations with market may dramatically change when market has a significant drawdown. Protection is not working when it is especially needed (2008 financial crisis)

Drawdown: Definition

$\{r_t\}_{1\le t\le T}$ = a sample path of scalar returns of some instrument

$\{w_t\}_{1\le t\le T}$ = vector of uncompounded cumulative returns,

$$w_t = \sum_{\nu=1}^{t} r_\nu \ , \quad 1 \le t \le T$$

$\{d_t\}_{1\le t\le T}$ = vector of drawdowns,

$$d_t = \max_{1\le \nu\le t} \{w_\nu\} - w_t \ , \quad 1 \le t \le T$$

- For every time moment t the **drawdown d_t is the difference between the previous maximum cumulative return and the current cumulative return**.

Drawdown: Example

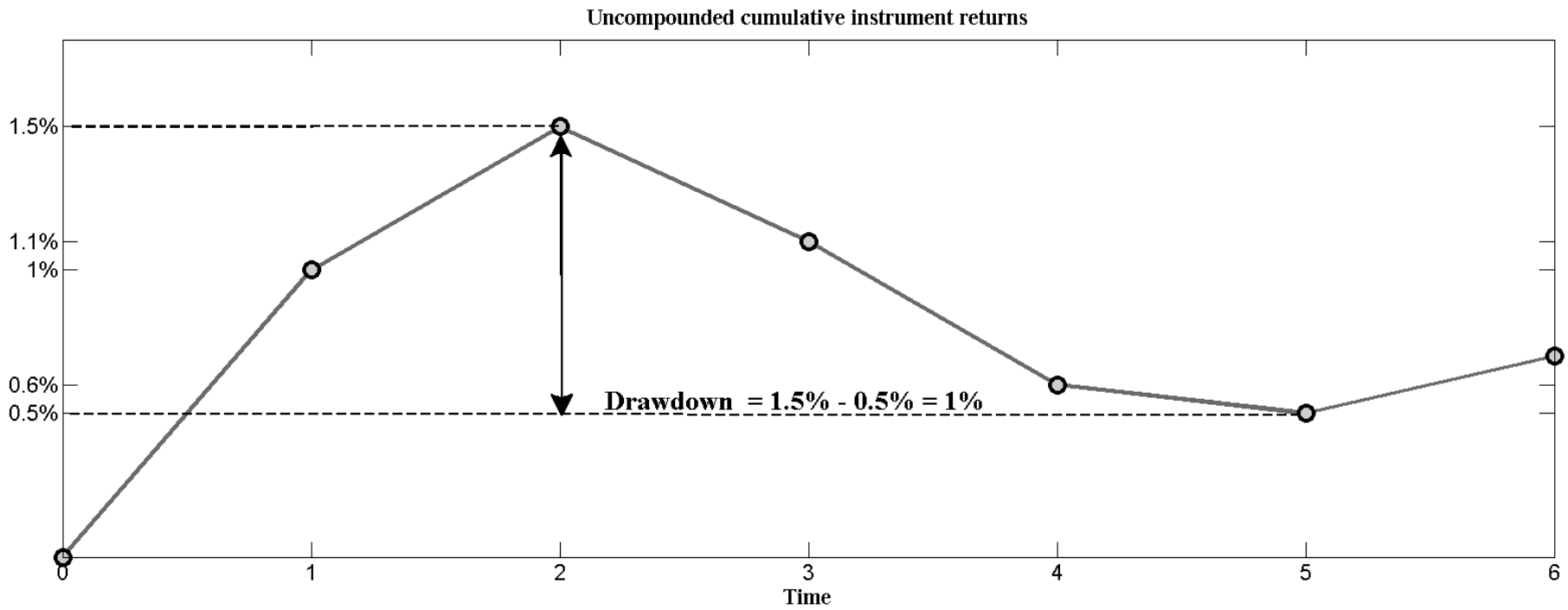


Figure: Solid line = uncompounded cumulative rate of return (at time t is the sum of rates of return over periods $1, \ldots, t$). For $t = 5$, $w_5 = 0.5\%$, whereas the maximum of w_t over time moments preceding $t = 5$ occurs at $t = 2$ with $w_2 = 1.5\%$. Consequently $d_5 = 1.5\% - 0.5\% = 1\%$. Maximum drawdown over time period $[0, 6]$ occurs at $t = 5$.

Drawdown Betas

- Drawdown Betas = show performance of an instrument when market is in drawdown
- Two variants of Drawdown Beta:
  1. CDaR Beta (earlier variant)
  2. ERoD Beta (recently developed)
- Instrument with negative Drawdown Beta generate positive return when market is in drawdown (at least in-sample)
- Drawdown Betas are obtained from necessary conditions of extremum for drawdown portfolio optimization problems (similar to the Standard Beta)
- Drawdown Betas may have very different values compared with Standard Beta (do NOT confuse with so called Downside Beta, which has values close to Standard Beta)

Drawdown Betas: Simplified Explanation

- Conditional Drawdown-at-Risk (CDaR) Beta:

$$\frac{average\ instrument\ losses\ over\ x\,\%\ worst\ case\ market\ drawdown\ periods}{average\ market\ losses\ over\ x\,\%\ worst\ case\ market\ drawdown\ periods}$$

- Expected Regret of Drawdown (ERoD) Beta:

$$\frac{average\ instrument\ losses\ during\ market\ drawdowns\ exceeding\ threshold}{average\ market\ drawdowns\ exceeding\ threshold}$$

Drawdown Betas: Formal Definition

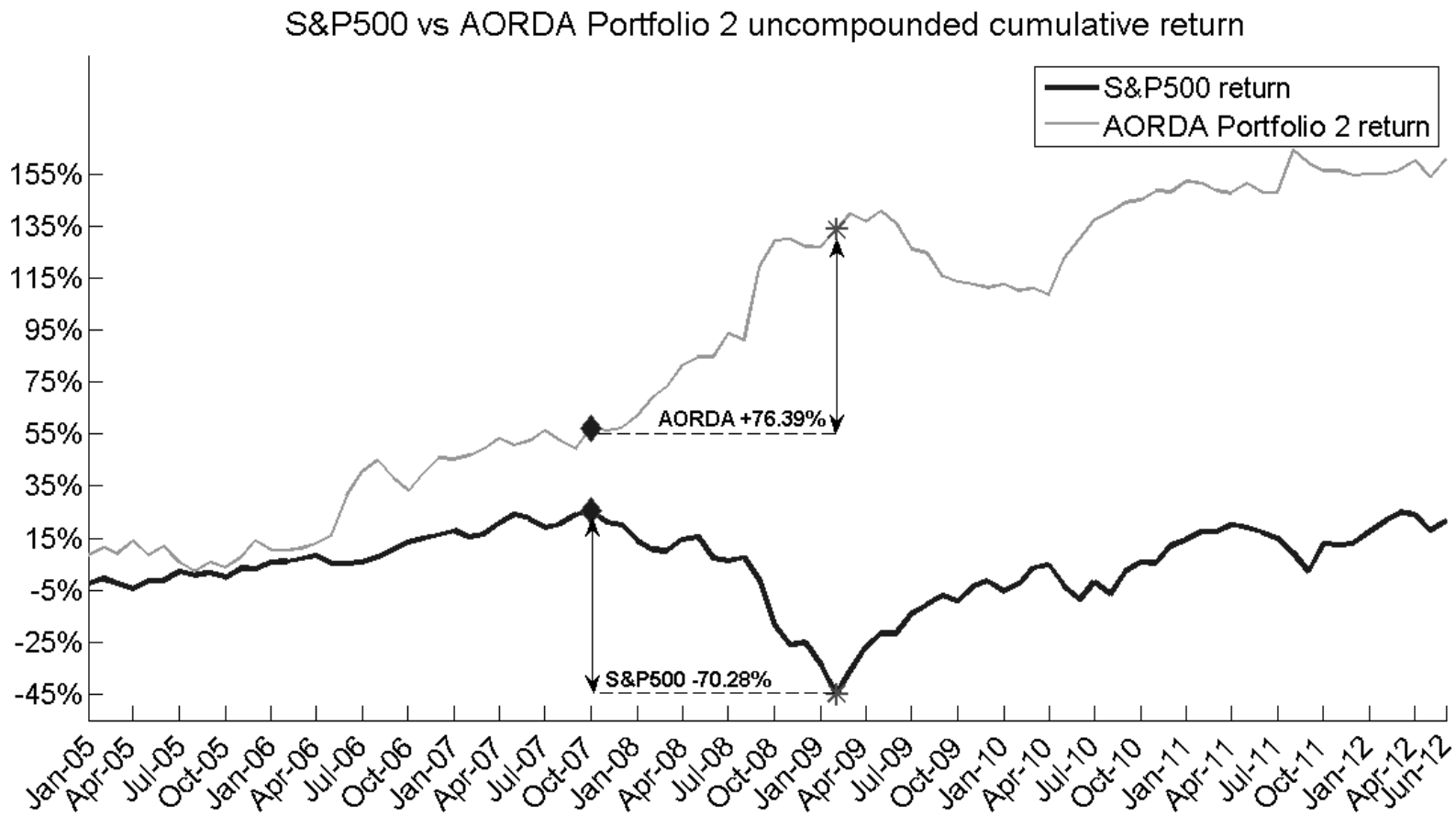
- CDaR Beta (Zabarankin, Pavlikov, Uryasev (2014)):

$$\beta^i_{CDaR} = \frac{\sum_{s=1}^{S}\sum_{t=1}^{T} p_s q^{\star}_{st}(w^i_{s,\tau(s,t)} - w^i_{st})}{CDaR_\alpha(w^M)}$$

- ERoD Beta (Ding and Uryasev (2021)):

$$\hat{\beta}^i_{ERoD} = \frac{\frac{1}{T}\sum_{s=1}^{S}\sum_{t=1}^{T} p_s q^{\star}_{st}(w^i_{s,\tau(s,t)} - w^i_{st})}{\tilde{E}_\epsilon(w^M)}$$

CDaR Beta: Maximum Drawdown Example



- $\text{CDaR}_{\alpha=1}$ Beta = MaxDD Beta $= 76.39\%/(-70.28\%) = -1.09$

Betas for DOW30 Stocks: 15-year Period

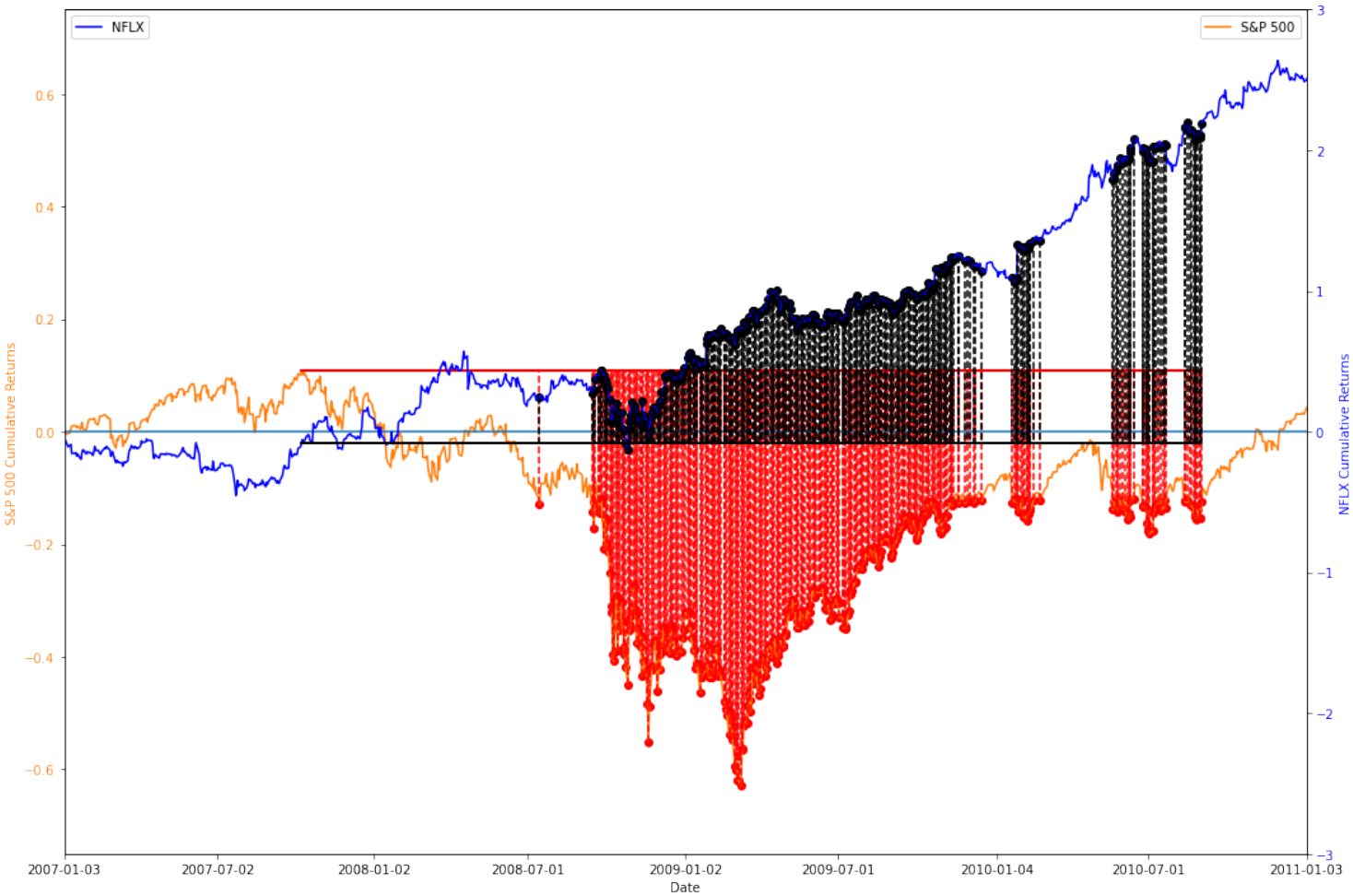
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	Downside
AAPL	-0.606	-0.062	0.98	1.024
AMGN	-0.094	-0.201	0.856	0.725
AXP	0.731	1.037	1.375	1.45
BA	1.321	1.492	1.017	1.254
CAT	0.798	1.003	1.183	1.106
CRM	-0.148	-0.257	1.428	1.153
CSCO	1.036	0.924	1.142	1.001
CVX	0.132	0.146	1.007	1.103
DIS	0.221	0.445	0.927	1.031
GS	0.877	0.556	1.334	1.384
HD	0.145	0.203	1.022	0.969
HON	0.803	1.002	0.97	1.05
IBM	0.022	0.008	0.812	0.794
INTC	0.333	0.51	0.973	1.006
JNJ	0.003	0.076	0.577	0.571
JPM	-0.534	-0.678	1.317	1.447
KO	-0.122	0.172	0.52	0.593
MCD	-0.826	-0.389	0.59	0.682
MMM	0.685	0.647	0.794	0.823
MRK	0.661	0.903	0.779	0.746
MSFT	-0.074	0.256	1.02	0.968
NKE	-0.714	-0.167	1.015	0.956
PG	0.144	0.306	0.666	0.583
TRV	-0.32	-0.228	0.907	1.055
UNH	0.593	0.922	0.72	0.985
VZ	0.391	0.535	0.758	0.628
WBA	0.339	0.385	0.816	0.697
WMT	-0.57	-0.581	0.633	0.503

Do Drawdown Betas Hold the Value over Time?

	$ERoD_{0+}$-Beta	$CDaR_{0.9}$-Beta	Standard-Beta
DOW30	0.275	0.515	0.676
S&P 100	0.305	0.449	0.645
S&P 500	0.074	0.293	0.577

- Correlation coefficients of ERoD, CDaR and Standard Betas between two 7-years periods for DOW30, SP100, and SP500 stocks.

Drawdown Betas (15-years): Netflix (NFLX)



- $CDaR_{0.9}$ Beta = **-2.388** based on largest 10% SP500 drawdowns
- Standard Beta = **0.85** based on monthly SP500 returns

Conclusion

- Drawdown Betas are obtained from necessary optimality conditions for drawdown portfolio optimization problems
- Drawdown Betas may have quite different values compared to Standard Beta
- Drawdown Betas hold value across time reasonably well for large stocks (DOW30, SP100)
- Drawdown Betas can be used for constructing portfolios with controlled drawdown
- Zero Drawdown Beta constraints can be imposed in the portfolio optimization problems (similar to Zero Standard Beta constraint)
- Drawdown Beta website shows CDaR, ERoD, and Standard Betas as well as other characteristics for SP500 stocks: http://qfdb.ams.stonybrook.edu/index_SP_10.html
- Quantitative Finance Program at Stony Brook University includes a Case Study Course containing material related to Drawdown Betas