CASE STUDY: VaR vs Difference of CVaRs Minimization (VaR, CVaR, Linear)

Background

This case study demonstrates that minimization of difference of CVaRs may provide approximately the same result as minimization of VaR. We approximate VaR_α (with confidence level α) by average of VaRs, with confidence levels in interval $[\alpha_1, \alpha_2]$. This interval contains confidence level α. It can be proved that the average of VaRs in interval $[\alpha_1, \alpha_2]$ equals

$$\frac{(1-\alpha_1)*CVaR_{\alpha_1} - (1-\alpha_2)*CVaR_{\alpha_2}}{\alpha_2 - \alpha_1} = \frac{1-\alpha_1}{\alpha_2-\alpha_1} CVaR_{\alpha_1} - \frac{1-\alpha_2}{\alpha_2-\alpha_1} CVaR_{\alpha_2} \,. \qquad \text{(CS.1)}$$

In this case study, α = 0.95, α_1 = 0.94, and α_2=0.96. Therefore,

$$\frac{1-\alpha_1}{\alpha_2-\alpha_1} = \frac{1-0.94}{0.96-0.94} = 3, \text{ and } \frac{1-\alpha_2}{\alpha_2-\alpha_1} = \frac{1-0.96}{0.96-0.94} = 2\,.$$

Here we considered the credit risk portfolio optimization problem for portfolio of clusters of retail loans from the case study "VaR Optimization Retail Portfolio of Bonds", see

http://www.ise.ufl.edu/uryasev/research/testproblems/financial_engineering/var-optimization-retail-portfolio-of-bonds/

Notations

I = number of instruments (clusters) in the portfolio; $i=\{1,\ldots,I\}$ index of instrument in the portfolio;
J = number of scenarios, $j=\{1,\ldots,J\}$ index of scenarios;
$x = (x_1,\ldots,x_I)$ = vector of exposures (weights) of instruments $i=1,\ldots I$;
l_i = lower bound on exposure to instrument i ;
u_i = upper bound on exposure to instrument i ;
r_i = rate of return of i-th instrument;
r_{ij} = rate of return of $\boldsymbol{i}$-th instrument under the risk scenario j ;
$\boldsymbol{r}_j = (r_{1j},\ldots,r_{Ij})$ =vector of rates of returns of instruments $i=1,\ldots I$ under the scenario j ;
$L(x,\boldsymbol{r}_j) = -\sum_{i=1}^{I} r_{ij} x_i$ = loss under scenario j ;
α = confidence level in VaR;
α_1 = lower confidence level in CVaR ($\alpha_1 < \alpha$);
α_2 = upper confidence level in CVaR ($\alpha_2 > \alpha$);
R = target level of rate of return of the portfolio.

Optimization Problem 1

minimizing VaR

$$\min_x VaR_\alpha(L(x,\boldsymbol{r})) \qquad \text{(CS.2)}$$

subject to

linear constraint

$$\sum_{i=1}^{I} r_i \; x_i \geq R \tag{CS.3}$$

upper/lower bounds on variables

$$l_i \leq x_i \leq u_i \,,\; i = 1, \ldots, I \tag{CS.4}$$

Optimization Problem 2

minimizing difference of CVaRs

$$\min_x \left[\frac{1-\alpha_1}{\alpha_2 - \alpha_1} CVaR_{\alpha_1}(L(x, \mathbf{r})) - \frac{1-\alpha_2}{\alpha_2 - \alpha_1} CVaR_{\alpha_{12}}(L(x, \mathbf{r})) \right] \tag{CS.5}$$

subject to

linear constraint

$$\sum_{i=1}^{I} r_i \; x_i \geq R \tag{CS.6}$$

upper/lower bounds on variables

$$l_i \leq x_i \leq u_i \,,\; i = 1, \ldots, I \tag{CS.7}$$