CASE STUDY: Production Planning (fxchg, linear)

Background

This case study demonstrates an optimization setup and relevant graphs for a single item capacitated lot size model. For a finite time horizon T, there is demand for a single item in each production period. This demand must be satisfied by the production in that period or by inventory from previous periods, that is, no backlogging is allowed. The production level cannot exceed a certain capacity limit. Two kinds of costs are considered, production cost and holding cost. Production, if initiated in a certain period, requires initial setup cost. We are trying to find a feasible production plan that minimizes total costs.

We have solved the problem with two solvers, VAN and CARGRB. The CARGRB solver uses GUROBI and can be used only if GUROBI is installed. Although, for this problem both solvers give optimal solution, typically, CARGRB solver is a preferred solver for the problems involving Fixed Charge function.

References

- Chen, H-D., Hearn, D. W., Lee, C-Y. (1994): A new dynamic programming algorithm for the single item capacitated dynamic lot size model, *Journal of Global Optimization*, Vol.4, No. 3/April

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- Chen, H-D., Hearn, D. W., Lee, C-Y. (1995): Minimizing the error bound for the dynamic lot size model, *Operations Research Letters*, 17, 57-68

- Atamturk, A., Munoz, J. C. (2004): A study of the lot-sizing polytope, Mathematical Programming 99, 443-465

Notation

$\vec{x} = (x_1, \dots, x_T)$, decision vector, x_t = production level in period t

$\vec{I} = (I_1, \dots, I_T)$, decision vector, I_t= inventory level at the end of period t

T= total number of periods

p_t = unit production cost in period t

s_t= set up costs for period t

d_t= demand in period t

c_t=production capacity in period t

$\overrightarrow{lb}$ = vector of lower bounds on production level

$\overrightarrow{ub}$ = vector of upper bounds on production level

Fixcharge Positive Function is defined as follows:

$$\textbf{fxchg\_pos}(\vec{x},\textbf{w}) = \sum_{t=1}^{T} s_t g(x_t, w), w \geq 0 \text{ threshold}, \; g(x_t, w) = \begin{cases} 1, x_t \geq w \\ 0, x_t < w \end{cases}$$

Optimization Problem

Minimizing total costs

$$\min_{\vec{x},\vec{I}} \left\{ \sum_{t=1}^{T} (p_t x_t + I_t) + fxchg\_pos(\vec{x}, w) \right\} \qquad \text{(CS.1)}$$

subject to

Constraint on production demand

$$I_{t-1} + x_t - I_t = d_t, \quad t = 1, \dots, T \qquad \text{(CS.2)}$$

Bounds on positions

$$I_0 = 0 \quad I_t \geq 0 \quad t = 1, \dots, T \qquad \text{(CS.3)}$$

$$lb_t \leq x_t \leq ub_t \quad t = 1, \dots, T \qquad \text{(CS.4)}$$